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Remote Entanglement of Solid-State Qubits

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Abstract

Entanglement is a quantum phenomenon involving two or more particles, where the particles are described by a single wavefunction that cannot be separated into sub-particle states. First, we present one of the possible platforms for entangling remote qubits using the single-photon protocol. It combines quantum nodes that are based on nitrogen-vacancy centers in bulk diamond by establishing remote entanglement between their electron spin qubits. Then, we describe different quantum network protocols for generating entanglement. Finally, we present recent experiments demonstrating multinode operation and metropolitan-scale remote entanglement between NV-center-based quantum nodes.

Contents

1 Introduction

Quantum physics has become an integral part of understanding the basic building blocks of matter and has fundamentally changed our understanding of nature, leading to some of the largest technological advancements. It can be used to construct more secure communication channels or networks. One of the platforms for quantum networks is remote entanglement across multiple distant quantum nodes. They rely on three important properties of quantum systems. Quantum superposition allows a system to occupy multiple states at the same time. Quantum entanglement generates nonclassical correlations between the states of distinct subsystems. Quantum measurement collapses the superposition into a single state. Quantum technologies are currently interesting both technologically and scientifically [1, 2].

In this seminar, we focus on remote entanglement in solid-state qubits in nitrogen-vacancy centers in diamonds. In the first section, we describe entanglement and its properties. Next, we describe the matter in our quantum nodes and how we generate entanglement between them. Lastly, we present experiments from Ref. [3], where they heralded entanglement of solid-state qubits on a metropolitan-scale, and Ref. [2], where they realized a multinode quantum network of remote solid-state qubits.

2 Entanglement

Entanglement is a quantum phenomenon that has no classical counterpart. If two (or more) particles are entangled, they will be more strongly correlated than classical physics allows. Particles are described by a single wavefunction, which cannot be separated into sub-particle states. It plays a key role in various implementations of quantum computation, quantum communication, quantum key distribution, cryptography, teleportation, etc. For its application, the fundamental unit is a qubit, which can be realized using any general two-level quantum system [4].

We can write any single qubit state as: $|\psi\rangle = \sum_i c_i |\phi_i\rangle$, where $\sum_i |c_i^2| = 1$ and $\{|\phi_i\rangle\}$ form an orthonormal basis in the Hilbert space. A two-qubit system is described by the tensor product of those two otherwise independent systems: $|\psi_{AB}\rangle = |\psi\rangle_A \otimes |\psi\rangle_B$. The tensor product combines two Hilbert spaces of individual subsystems into a four-dimensional space. For example, we can write local states as general pure states: $|\psi\rangle_A = a_0 |0\rangle + a_1 |1\rangle$ and $|\psi\rangle_B = b_0 |0\rangle + b_1 |1\rangle$, where $|0\rangle$ and $|1\rangle$ represent two levels of the systems and $a_0,\ a_1,\ b_0,\ b_1$ probability amplitudes. Taking the tensor product of the two states, we arrive at the following two-qubit state:

$$\begin{aligned} |\psi\rangle = |\psi\rangle_A \otimes |\psi\rangle_B = (a_0 |0\rangle_A + a_1 |1\rangle_A) \otimes (b_0 |0\rangle_B + b_1 |1\rangle_B) = \\ = c_1 \ |0\rangle_A |0\rangle_B + c_2 \ |0\rangle_A |1\rangle_B + c_3 \ |1\rangle_A |0\rangle_B + c_4 \ |1\rangle_A |1\rangle_B \ , \end{aligned} \tag{1}$$

where $c_1,\ c_2,\ c_3,\ c_4$ are constants. We obtain a superposition of four possible two-particle states, each with its own probability amplitude. In such states, we can see the state of each subsystem separately. We call them separable states. However, not all possible states are product states, and we can have a statistical mixture of states [1].

A state is entangled if it is not separable, meaning that we cannot write it as a product state like Eq. 1. Since particles cannot be described separately, the results of measurements on an individual particle are correlated. This means that if one particle is measured, accurate predictions can be made about the state of the other entangled particle. The most common examples are the so-called *Bell states*:

$$|\psi^{\pm}\rangle = \frac{1}{\sqrt{2}} (|0\rangle_A |1\rangle_B \pm |1\rangle_A |0\rangle_B) \tag{2}$$

$$|\phi^{\pm}\rangle = \frac{1}{\sqrt{2}} (|0\rangle_A |0\rangle_B \pm |1\rangle_A |1\rangle_B) \tag{3}$$

which are maximally entangled, meaning that the states have a maximum correlation with each other. They form a basis for all two-qubit states.

Another example of maximally entangled states are the so-called *GHZ states*:

$$|GHZ\rangle = \frac{1}{\sqrt{2}} \left(|0\rangle^{\otimes n} \pm |1\rangle^{\otimes n} \right) , \tag{4}$$

where we can generalize them for n qubits, where $n \geq 3$ [1, 4].

To check the quality of the entangled state, one needs to calculate its characteristics. One of the key characteristics is fidelity F, which tells us how close the real state is to the target state. Its bounds are $0 \leq F \leq 1$, and the higher the fidelity, the greater the overlap with the target state [4].

3 Nitrogen-Vacancy Centers

The nitrogen-vacancy (NV) center is a photoluminescent point defect in diamond. They are imperfections in the crystal at single lattice points. It consists of a nearest-neighbor pair of a nitrogen atom, which substitutes for a carbon atom, and a lattice vacancy [5].

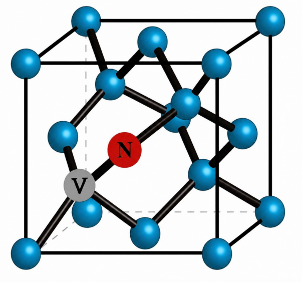

Figure 1: Simplified atomic structure of the NV center [5].

Natural NV centers can be formed in diamonds but are randomly oriented within a diamond crystal, while ion implantation techniques can enable their artificial creation in predetermined positions. There are two main routes for the fabrication of diamonds containing NV centers. Nitrogen can be implanted post-growth or incorporated as a dopant during growth, depending on the desired number of NV centers. We can get two charged states of this defect: neutral NV^0 and negative NV^-. The neutral state is not generally used for quantum technology and can be converted into NV^- by changing the Fermi level position with an external voltage [6].

They can be used for applications across quantum technologies, condensed matter physics, and even biosensing. The most notable application has been as a quantum sensor for various physical quantities, such as magnetic and electric fields, temperature, pressure, and strain. We can also initialize them as qubits and use them for quantum algorithms and networks [5, 6].

3.1 Generating Entanglement Between NV Centers

To generate remote entanglement between quantum nodes, a single-photon protocol is used. Since NV centers in diamond provide stable, long-lived qubits that can be initialized, manipulated, and read out at room temperature, they are ideal for such implementations. We use their negative charge state, where an additional electron from the environment is trapped and a spin-1 system is formed [7].

For our qubit subspace, we use the ground state $|g\rangle$, which is a spin-triplet and has three sublevels denoted by $|0\rangle = |g, m_s = 0\rangle$ and $|1\rangle = |g, m_s = \pm 1\rangle$. The $m_s = \pm 1$ sublevels are degenerate and lie slightly higher due to their spin-spin interaction. We can drive the transition between the two ground states with a microwave generator and change the population of the sublevels within the ground state. We additionally use an excited state $|e\rangle$, which has a similar structure, being also a spin-triplet. There are also two intermediate states $|s\rangle$ that form one of the pathways for the NV center to return to its ground state. If we illuminate the system with a laser that is resonant with the transition from the ground state to the excited state, we excite the NV center with a spin-conserving transition. This means that the only transitions that are allowed are from $|g, m_s = 0\rangle$ to $|e, m_s = 0\rangle$ and from $|g, m_s = \pm 1\rangle$ to $|e, m_s = \pm 1\rangle$. If the system is excited, it must relax back to the ground state. It can undergo a direct spin-conserving radiative decay from the excited state to the ground state. In this process, red photons with a specific wavelength, directly proportional to the energy difference between levels, are emitted. For the NV center $\lambda = 637$ nm. An alternative pathway includes metastable intermediate states and a non-radiative decay to the ground state. In this process, photons are not emitted. Instead, the energy is released as heat through molecular vibrations and motion [5].

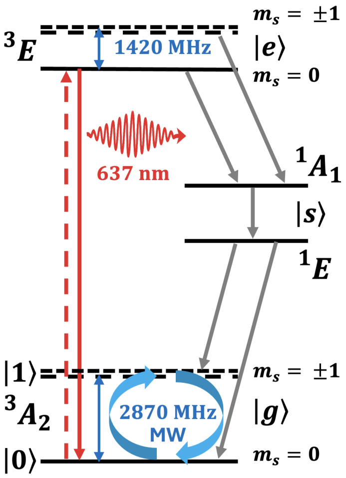

Figure 2: Schematic energy level structure of the NV center.

Additionally, we can selectively address the transition from the ground state to the excited state, which is necessary for entanglement generation. Since $m_s = 0$ and $m_s = \pm 1$ are not degenerate, and the splitting between them in the ground and excited states is different, the optical transitions occur at slightly different optical frequencies. One possibility is to use a narrowband laser, where you can choose the desired transition. This works especially well at cryogenic temperatures [7]. Another possibility is to use a magnetic field, which increases spectral separation [2].

For the single-photon protocol, we need two remote stationary qubits that generate qubit-photon entangled states and a detection of a single photon. Lets start with one NV center that is prepared with microwaves in the superposition state $|\psi\rangle = \sqrt{\alpha}\,|0\rangle + \sqrt{1-\alpha}\,|1\rangle$, where $|0\rangle$ and $|1\rangle$ represent the ground states in the NV center, and α denotes the population in $|0\rangle$. An optical pulse resonant only with the $|0\rangle$ state is applied. The optical excitation is engineered such that spontaneous emission occurs only to the $|0\rangle$ state, producing a photon, while the $|1\rangle$ state remains optically dark. After the optical interaction, the NV center and the photon become entangled, giving us the state $|\psi\rangle = \sqrt{\alpha}\,|0\rangle\,|1\rangle_\gamma + \sqrt{1-\alpha}\,|1\rangle\,|0\rangle_\gamma$, where $|1\rangle_\gamma$ means that the photon exists and $|0\rangle_\gamma$ means that there is no photon. If a photon is detected, only the first term survives, and the state of the NV center collapses into $|0\rangle$. If no photon is detected, only the second term survives, and the state of the NV center collapses into $|1\rangle$. Now consider two NV center, where each is prepared in the superposition $|\psi\rangle_i = \sqrt{\alpha}\,|0\rangle + \sqrt{1-\alpha}\,|1\rangle$ where $i = A, B$. After the excitation, we get $|\psi\rangle \approx |11\rangle\,|0_A 0_B\rangle_\gamma + \sqrt{p}\left(|01\rangle\,|1_A 0_B\rangle_\gamma + |10\rangle\,|0_A 1_B\rangle_\gamma\right) + \mathcal{O}(p^2)$, where p is a constant. The only one-photon term is the second, meaning only NV center A or B emitted the photon. The photons from the two quantum nodes are then interfered on a beam splitter, which removes the which-path information and the detectors cannot tell which NV center emitted the photon. A single detector click preserves both possibilities coherently. The beam splitter closes the effective interferometer formed by the two quantum nodes with NV centers. It is followed by single-photon detection. Measurement of one photon after the beam splitter heralds the entanglement of the qubit states to $|\psi^\pm\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle \pm e^{i\theta}\,|10\rangle\right)$, with maximal fidelity of $F = 1-\alpha$, where the ± sign depends on which of the two detectors clicked, and θ is the optical phase difference between the two arms of the effective interferometer [7, 8].

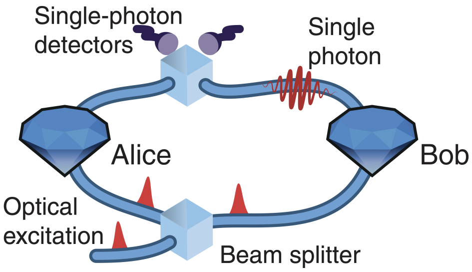

Figure 3: Sketch of two quantum network nodes [2].

The entanglement generation protocol operates in the weak-excitation regime, where the probability $p \ll 1$ of generating a detected photon per node per attempt is small. In this regime, the probability that neither quantum node emits is large, while the probability that exactly one quantum node emits is proportional to p, and the probability that both quantum nodes emit is proportional to p^2, making such events rare. It is achieved by reducing the optical excitation power of the laser pulses. In addition, the relative population of spin states that contribute to emission can be optimized via microwave preparation, improving the success probability and state fidelity.

In this protocol, the detection of one photon qubit conditions the formation of entanglement between the remote quantum nodes. Any experimental repetition where two photons are detected in different detectors is rejected. Since our model is not ideal, we have to consider possible imperfections that make different detection patterns look identical. The most prominent factors are photon loss, double excitation, and the presence of noise counts [3].

The first detection pattern we consider is a single photon. This is the case where one photon is emitted and one photon is detected. This is the ideal case, where one excitation occurs in one of the quantum nodes. The second detection pattern is when two photons are emitted, either both by the same quantum node (double excitation) or by the two quantum nodes separately, and both photons arrive at the beam splitter. As mentioned before, two-photon events are rejected. However, if both photons leave the beam splitter at the same port, we cannot discriminate between a two-photon event and a single-photon event. Photon loss gives us another detection pattern. When at least one photon is lost, the detection of the other photon will falsely herald entanglement. Lastly, we have noise photons, which can come from detector dark counts or stray light and can lead to false entanglement [7].

3.1.1 Postselected Entanglement

In this protocol, the qubits are measured directly after generating entanglement, and successful photon detection at the detectors is used in postprocessing to analyze entanglement generation [3]. Here, we run the experiment many times and record photon detection events. Afterwards, we select only detection patterns where one photon was detected. This means that we conclude that quantum nodes with NV centers were entangled, even though we didn't know it at that time.

The entanglement exists only in a post-selected subset. Its key feature is that it is probabilistic and requires classical knowledge of the measurement outcome. This process is compatible with quantum key distribution, but it does not allow for more advanced protocols since the state is not available for further use [9].

3.1.2 Fully Heralded Entanglement

In contrast to postselected entanglement generation, here "live" entangled states are delivered to the quantum nodes and can be further used [3]. We run the experiment and wait for the specific detection pattern to occur. If one photon is detected, we know immediately that the quantum nodes are entangled. If none or two photons are detected, we discard the attempt and try again in real time. The photon detectors play a key role, as their clicks are the heralding signal of successfully created entanglement between two qubits. The main benefit of this method is that we do not disturb the qubits and they remain untouched and entangled [9].

Live entanglement is a fundamental requirement for many future applications of long-range entangled states and is needed for entanglement swapping. It allows reliable quantum communication, quantum repeaters, and scalable architectures that would lead to a secure quantum network [3, 9].

4 Metropolitan-scale Entanglement

One of the key challenges in building a quantum network is scaling current laboratory experiments into metropolitan-scale quantum systems capable of real-world deployment. A quantum network system must be able to generate, store, and process quantum information on these larger scales. They require quantum nodes to operate fully independently. Since the paths in optical fibers will extend for tens of kilometers, photon loss becomes a critical parameter that must be mitigated. Advanced network applications require the heralded delivery of entanglement; the qubit systems must be able to store quantum information, while the network must support real-time feedback to the qubits upon successful entanglement generation.

Physicists from QuTech and Kavli Institute of Nanoscience at Delft University of Technology have succeeded in generating heralded entanglement of solid-state qubits between two independently operated quantum nodes separated by 10 kilometers (Fig. 4).

Each quantum node houses a chemical vapour deposition diamond with NV centers [6] that can generate entanglement. The ground state levels are additionally split using a small magnetic field of ≈ 3 mT, which is aligned with the NV axis. This allows for arbitrary qubit rotations with a microwave pulse frequency of ≈ 2.8 GHz, preparing a spin superposition state with amplitude $\alpha = 0.25$. Reson-

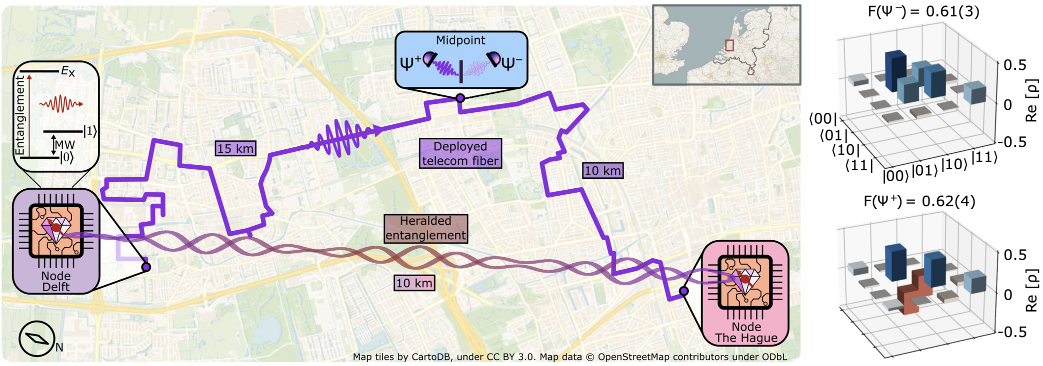

Figure 4: Cartographic layout of the distant quantum link and the route of the deployed fiber bundle and Bayesian estimation of density matrix using the post-selected entanglement generation scheme [3].

ant excitation is achieved with a narrowband 637 nm laser. Each quantum node is also equipped with a quantum frequency converter unit that converts the 637 nm NV photons down to the telecom L-band at 1588 nm. This is done to minimize photon loss in the fibers that connect each quantum node with the midpoint. To further reduce photon loss, they use a single-photon entanglement protocol that employs number basis encoding for the photons. The losses on the quantum channels are at 5.6 dB for Delft and 5.2 dB for The Hague. The midpoint consists of a beam splitter and superconducting nanowire single photon-detectors. It provides phase feedback, polarization control, and spectral filtering.

To determine the performance of the architecture, they monitored parameters and generated entanglement in postselection. In this case, both quantum nodes were in a single laboratory in Delft. It consisted of 540 rounds of entanglement generation, with every seventh entanglement round also containing optical pulses to maintain the stability of the local phase. To characterize the generated nonlocal states, their qubit-qubit correlation was measured in different readout bases. To check the fidelity of the states, they used Bayesian estimation of density matrices [10] and obtained $F(\psi^-) = 0.61$ and $F(\psi^+) = 0.62$. They achieved an entanglement generation rate of 0.48 Hz for a 20-ns window, which is equal to the success probability per attempt of 7.1×10^{-6} .

They also generated fully heralded entanglement over the described deployed quantum link. For this application, they applied the refocusing echo pulse to the qubits to preserve their qubit states with high fidelity while waiting for the heralded signals to return and be processed. The photons traveled to the midpoint in about 52 and 73 μs from The Hague and Delft, respectively. The entanglement generation runs were automatically repeated by the quantum nodes until a successful heralding signal was received from the midpoint. Once it was received, the system jumped to a different control sequence in the protocol. The delivered entangled states had a fidelity of $F(\psi^-) = 0.534$. Here, they achieved an entanglement generation rate of 0.022 Hz for a predefined 15-ns window. The reduction in rate is mainly due to the added communication delay needed for the heralding signal to travel to the quantum nodes [3].

5 Multinode Quantum Network

Another challenge in realizing a quantum network is sharing entanglement across multiple quantum nodes. For a multinode quantum network, we need multiple high-performance quantum nodes that include a communication qubit with an optical interface, as well as an efficient memory qubit for storage and processing.

It also requires that each elementary link is phase-stabilized independently. Multinode quantum networks are capable of running two key quantum network protocols: entanglement distribution and entanglement swapping [11]. These protocols are generated in a heralded fashion and deliver the final states ready for further use.

Physicists from QuTech and Kavli Institute of Nanoscience at Delft University of Technology have succeeded in generating heralded entanglement of solid-state qubits between three quantum nodes in two separate laboratories.

The network is composed of three spatially separated quantum nodes, labeled Alice, Bob, and Charlie. Each quantum node houses an NV center electronic spin as a communication qubit. Node Bob uses an additional carbon-13 nuclear spin as a memory qubit. The quantum nodes are connected through optical fibers for quantum signals, as well as classical communication channels for synchronizing control operations and relaying heralding signals. For both links, they achieved a fidelity of the entangled Bell states exceeding $F = 0.8$, which is on par with the highest fidelity reported for this protocol for a single link. The entanglement rates are 9 and 7 Hz for links Alice-Bob and Bob-Charlie, respectively.

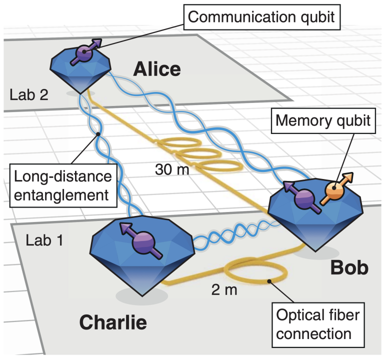

Figure 5: Schematic layout of the network [2].

An additional memory qubit in Bob's node is necessary to store the generated entangled states while new entanglement links are created. Carbon-13 nuclear spins are used because of their long coherence times, controllability, and isolation from the control drivers on the electronic qubit. Single Carbon-13 nuclear spins coupled to NV centers in diamond can exhibit coherence times exceeding 1 second using decoupling techniques. To improve its memory robustness, an additional magnetic field of 189 mT is used.

To establish a remote entangled state on each of the two links, the sequence depicted in Fig. 6 is used. After initializing the memory qubit, the first entanglement state is prepared on the link Alice-Bob. Once it is heralded, at Bob's node, deterministic quantum logic is used to swap the other half of the entangled state to the memory qubit. It is composed of rotational gates that are microwave (electron) and radio frequency (nuclear) pulses arranged to simulate a SWAP. It's followed by the generation of remote entanglement between the communication qubits of Bob and Charlie. In case of no success within the preset number of attempts, the full protocol is restarted. At the end, the two links each share an entangled state ready for further processing.

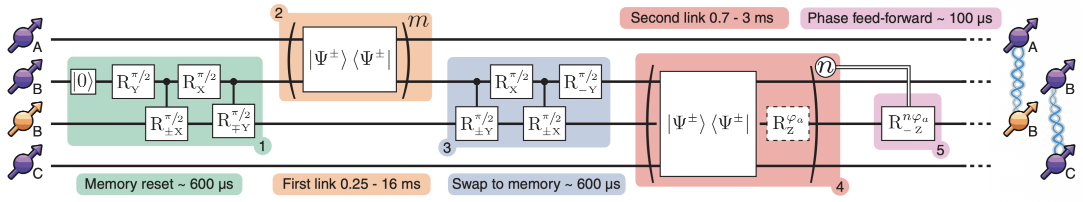

Figure 6: Circuit diagram displaying the experimental sequence used to establish entanglement on both elementary links [2].

The first protocol they performed was the generation of a multipartite entangled GHZ state across three nodes using entanglement distribution. First, they entangled the two qubits at Bob. This was followed by the measurement of the communication qubit in a suitably chosen basis. The remaining three qubits are projected into one of four possible GHZ-like states [11]. This protocol is able to achieve the delivery of the same GHZ state $|GHZ\rangle_{ABC} = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$ with a fidelity of $F = 0.538$ and a rate of 0.011 Hz.

The second protocol they performed was entanglement swapping, yielding an entangled state shared between the two nonconnected quantum nodes. They performed a Bell state measurement on two qubits at Bob's node, which projects them onto one of the four possible Bell states [11]. The unmeasured pair of particles then becomes entangled without any previous direct interaction between them. This protocol results in the delivery of the Bell state $|\phi^+\rangle_{AC} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ with a fidelity of $F = 0.551$ and a rate of 0.025 Hz [2].

6 Conclusions

Entanglement is a phenomenon that connects two particles to each other so that they become inseparable. This allows us to use one of the particles and its measurement to transfer information about the other particle, which remains with us. Utilizing NV centers in diamonds, we can generate heralded entanglement, which provides us with a live entangled state for future use. It is also compatible with the introduction of the memory qubit which prevents the entangled state from decohering. Entanglement is an important topic in quantum technology because it is the basis for the realization of a quantum network that would offer fast and secure information transfer. In Ref. [3], their architecture addresses key scaling challenges for bringing future quantum internet technology to a metropolitan scale. It establishes a generic platform that can be applied to other qubit platforms using photon interference to generate remote entanglement. The work in Ref. [2] establishes a key platform for exploring, testing, and developing multinode quantum network protocols and a quantum control stack. These results demonstrate the potential of NV-center-based platforms for scalable quantum networks and future quantum network applications that exploit quantum non-locality to solve coordination problems.

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