UNIVERSITY OF LJUBLJANA
FMF Faculty of Mathematics and Physics

Remote Entanglement of Solid-State Qubits

Seminar II – 2nd year, 2nd cycle

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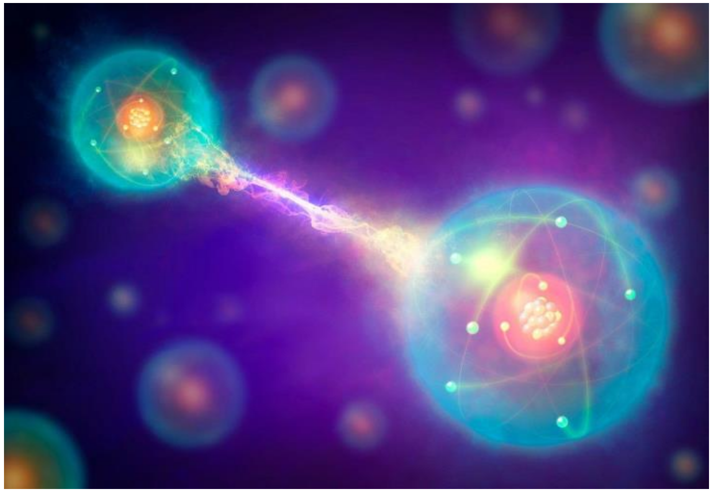
Why Do We Need Quantum Entanglement?

- key role:
  - quantum computation
  - quantum communication
  - quantum key distribution (QKD)
  - cryptography
  - teleportation
- secure communication channels → quantum internet

[1]

- challanges:
  - larger distances
  - multinode
- 3 important properties of the system
  - quantum superposition
  - quantum entanglement
  - quantum measurement
- no classical counterpart



[2]

Entanglement

- unit is a qubit
  - any general two-level quantum system
- single qubit state: $|\psi\rangle = \sum_i c_i |\phi_i\rangle$
- two-qubit system: $|\psi_{AB}\rangle = |\psi\rangle_A \otimes |\psi\rangle_B$
- general pure states: $|\psi\rangle_A = a_0|0\rangle + a_1|1\rangle$

  $|\psi\rangle_B = b_0|0\rangle + b_1|1\rangle$
- product state:

$$|\psi\rangle = |\psi\rangle_A \otimes |\psi\rangle_B = \left(a_0|0\rangle_A + a_1|1\rangle_A\right) \otimes \left(b_0|0\rangle_B + b_1|1\rangle_B\right) =$$

$$= c_1\,|0\rangle_A|0\rangle_B + c_2\,|0\rangle_A|1\rangle_B + c_3\,|1\rangle_A|0\rangle_B + c_4\,|1\rangle_A|1\rangle_B$$

- separable states
- statistical mixture of states

Entanglement

A state is entangled if it is not separable, meaning we cannot write it as a product state.

- measurements on individual particle are correlated
- examples:

  1) four *Bell states*

$$|\psi^{\pm}\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_A|1\rangle_B \pm |1\rangle_A|0\rangle_B\right)$$

$$|\phi^{\pm}\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_A|0\rangle_B \pm |1\rangle_A|1\rangle_B\right)$$

  2) *GHZ state*

$$|GHZ\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle^{\otimes n} \pm |1\rangle^{\otimes n}\right)$$

- fidelity: how close the real state is to the targeted state
  - $0 \leq F \leq 1$

Nitrogen-Vacancy (NV) Centers

- photoluminescent point defect in diamond
- types:
  - natural
  - implanted post-growth
  (HTHP – high-temperature high-pressure)
  - incorporated as a dopant during growth
  (CVD – chemical vapour deposition)
- two charged states: NV^0 and NV^-
- various applications

  → initialize them as qubits

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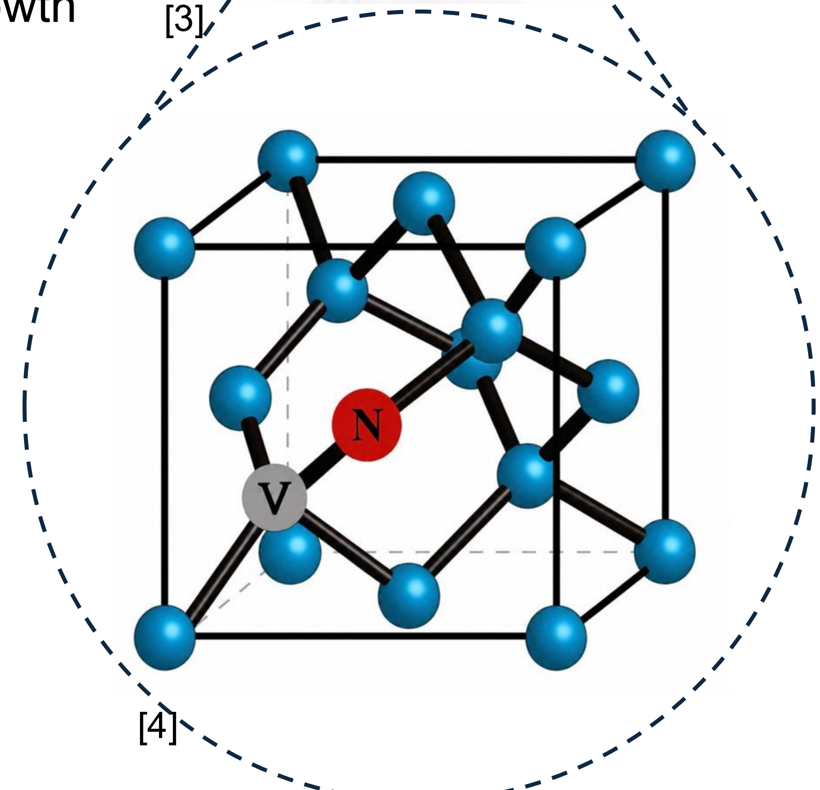

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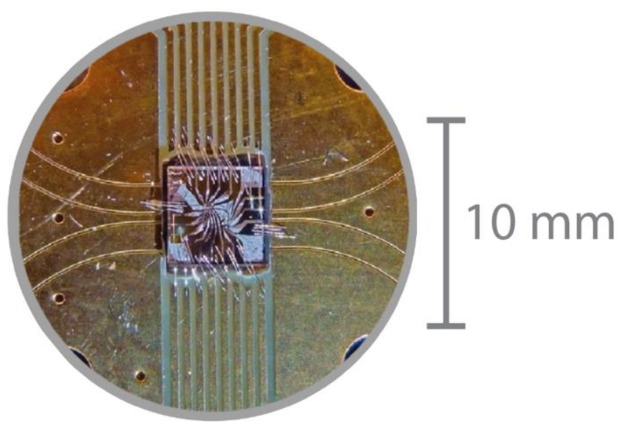

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Generating Entanglement Between NV Centers

○ single-photon protocol

= two remote stationary qubits + detection of a single photon

○ qubit subspace: $|g\rangle$

- $|0\rangle = |g, m_s = 0\rangle$
- $|1\rangle = |g, m_s = \pm 1\rangle$

○ excited state: $|e\rangle$

○ intermediate states: $|s\rangle$

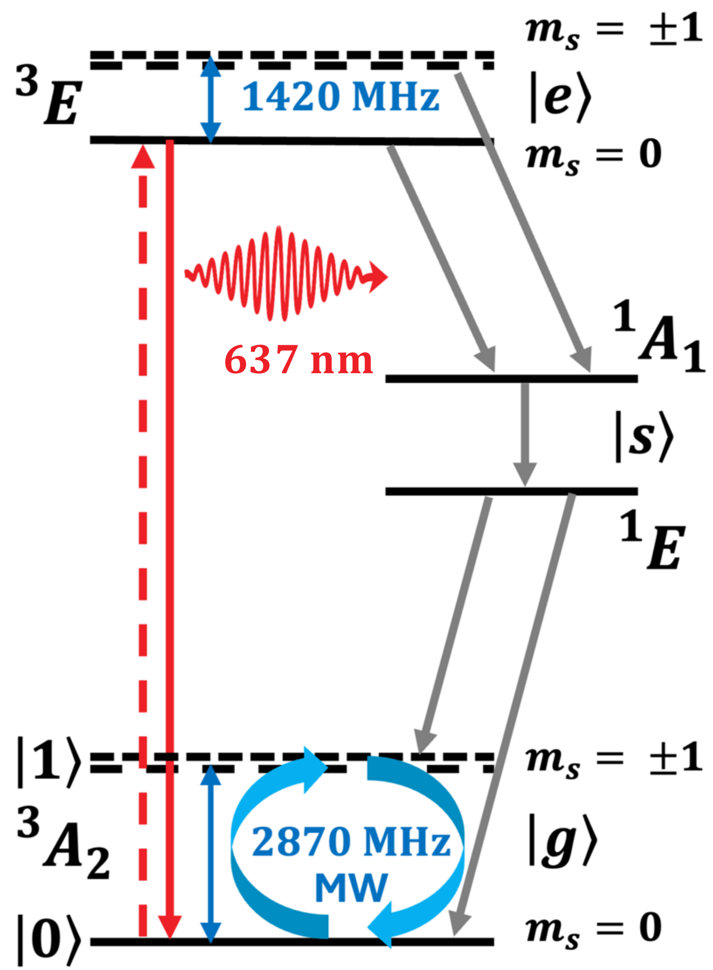
○ spin-conserving transition

○ spin-conserving radiative decay

$\Longrightarrow \lambda = 637\ \mathrm{nm}$ photon

○ selectively address the transition

- narrowband laser
- magnetic field



Generating Entanglement Between NV Centers

○ one NV center:

1) microwaves:

$$|\psi\rangle = \sqrt{\alpha}|0\rangle + \sqrt{1-\alpha}|1\rangle$$

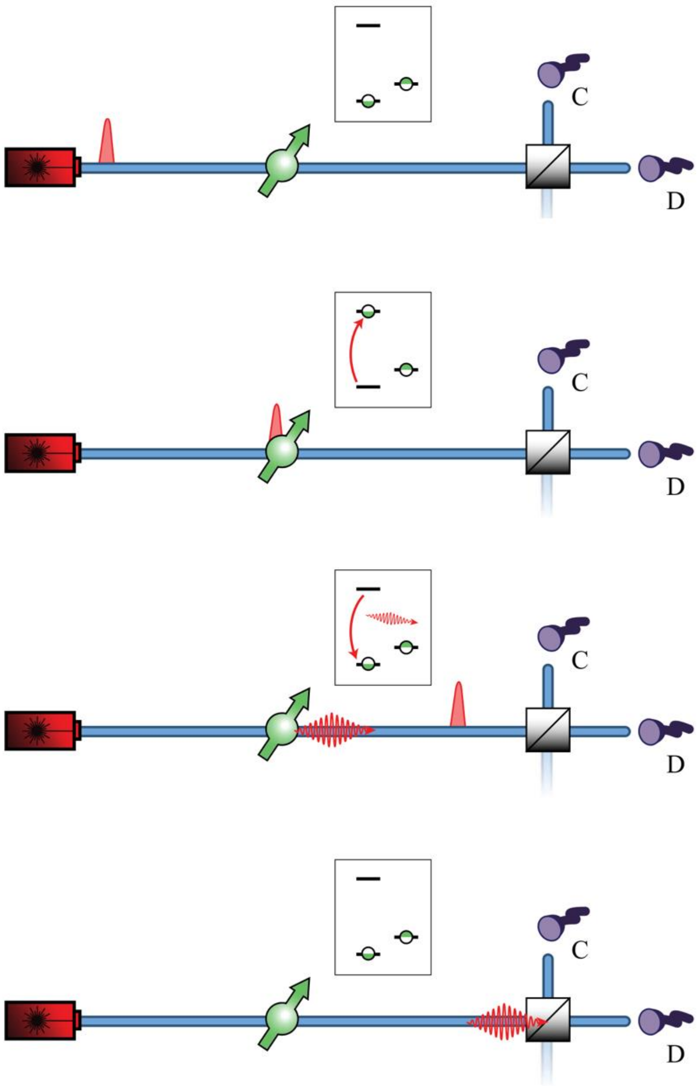
2) optical pulse resonant only with $|0\rangle$

→ NV center and photon entangled:

$$|\psi\rangle = \sqrt{\alpha}|0\rangle|1\rangle_\gamma + \sqrt{1-\alpha}|1\rangle|0\rangle_\gamma$$

3) photon detection → collaps of wavefunction:

$$|1\rangle_\gamma \Longrightarrow |\psi\rangle = |0\rangle$$
$$|0\rangle_\gamma \Longrightarrow |\psi\rangle = |1\rangle$$



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Generating Entanglement Between NV Centers

- two NV centers:

$$|\psi\rangle_i = \sqrt{\alpha}|0\rangle + \sqrt{1-\alpha}|1\rangle$$

- after excitation:

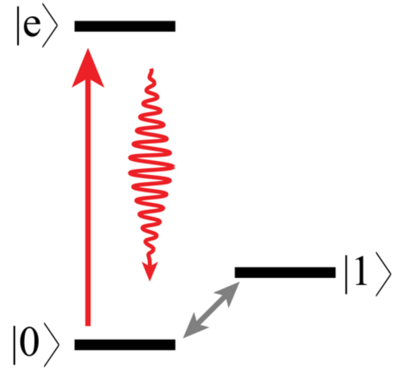
$$|\psi\rangle \approx |11\rangle|0_A 0_B\rangle_\gamma + \sqrt{p}\left(|01\rangle|1_A 0_B\rangle_\gamma + |10\rangle|0_A 1_B\rangle_\gamma\right) + \mathcal{O}(p^2)$$

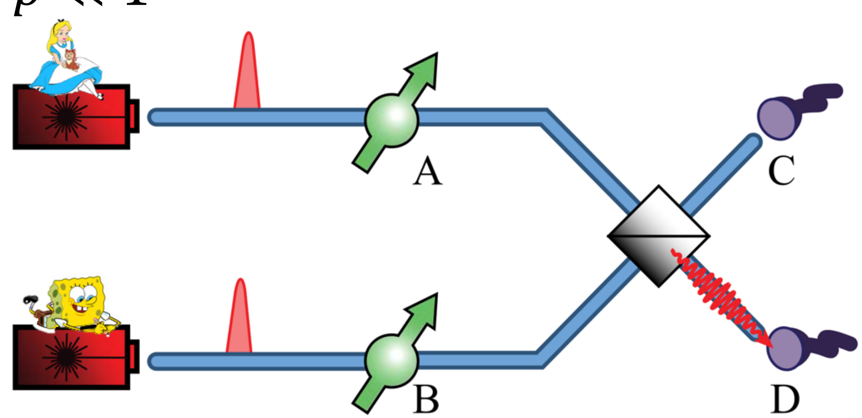
- beam splitter → removes the which-path information
- single detector click

⟹ heralds the entanglement:

$$|\psi^\pm\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle \pm e^{i\theta}|10\rangle\right)$$

- weak-excitation regime $p \ll 1$





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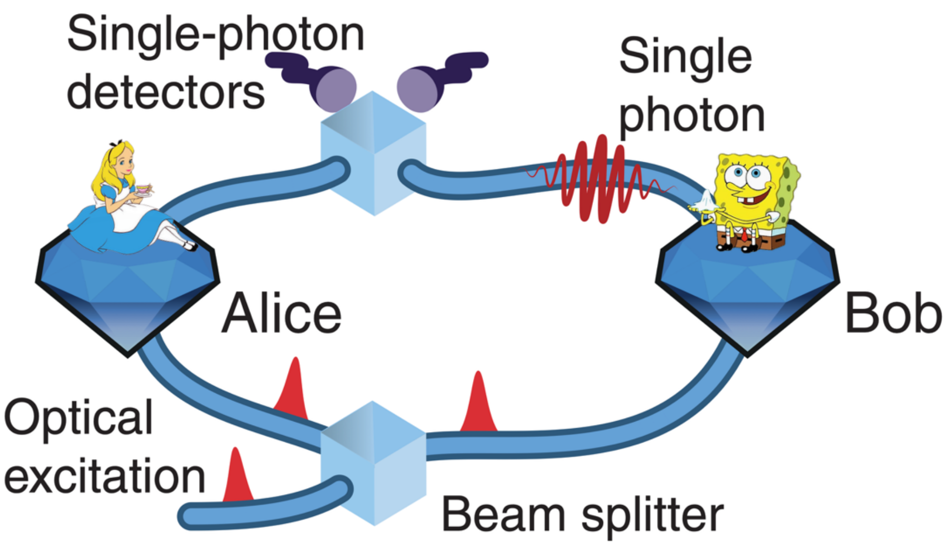
Generating Entanglement Between NV Centers

- condition for success: detection of one photon
- imperfections due to:
  - photon loss
  - double excitations
  - noise counts
- detection patterns:
  1) single photon
  2) two emitted photons
  3) at least one photon is lost
  4) noise photons



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Generating Entanglement Between NV Centers

1) Postselected Entanglement

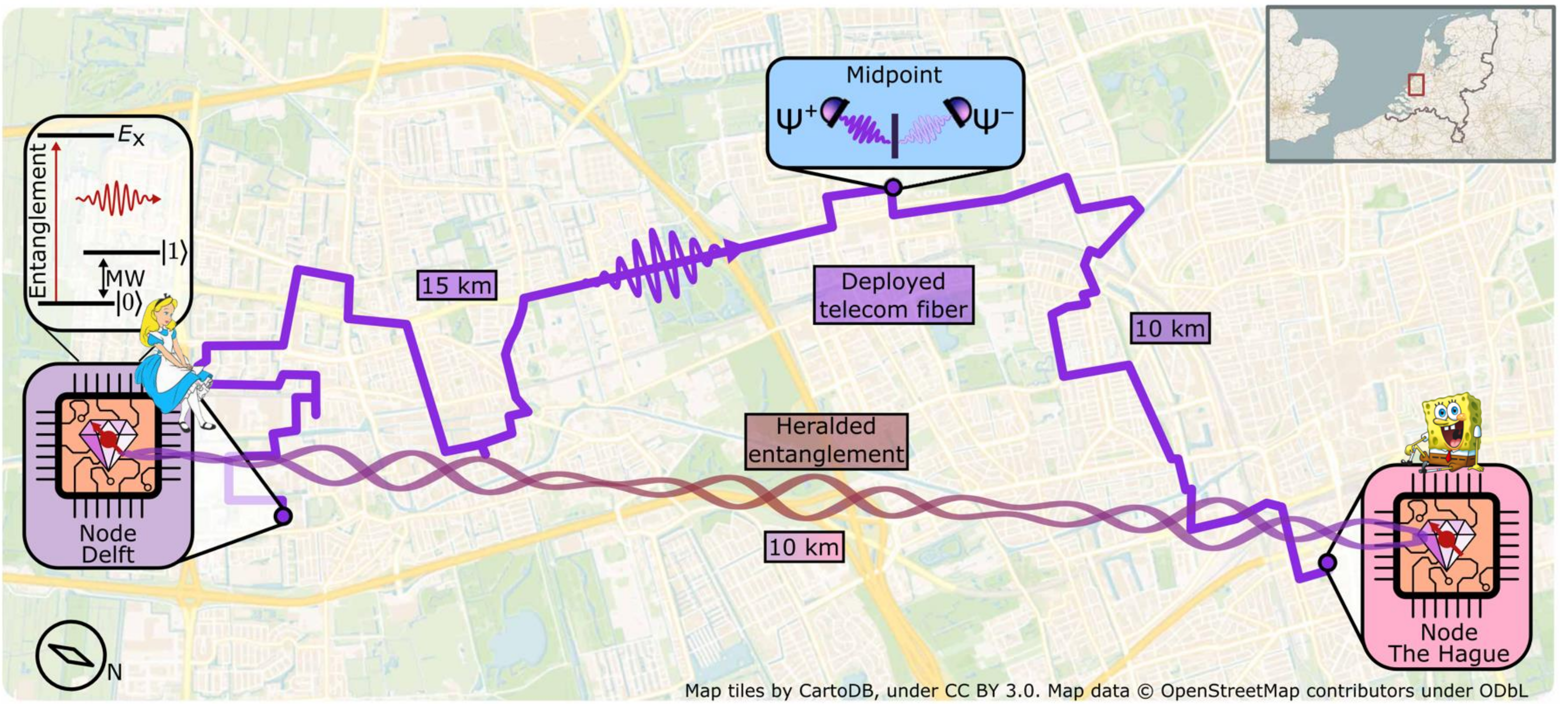
- measured directly after generating entanglement
- run and record
- requires classical knowledge of measurement outcome
- QKD

2) Fully Heralded Entanglement

- “live“ entangled states that can be used
- run and wait
- clicks are the heralding signal
- key component for many future applications

Metropolitan-Scale Entanglement

- operate fully independently
- photon loss
- Delft University of Technology, Netherlands
- separated by 10 km



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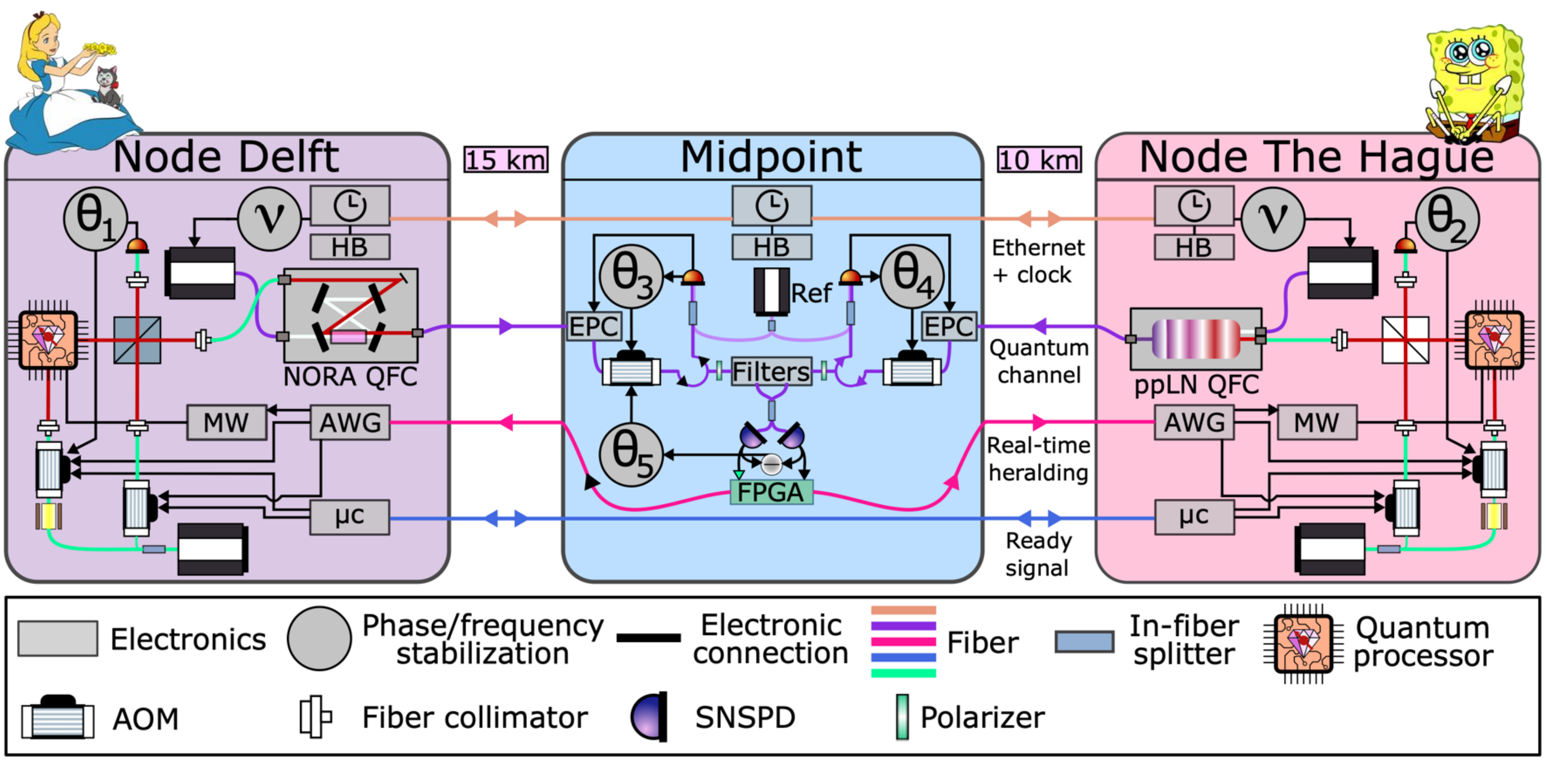
Metropolitan-Scale Entanglement

- CVD diamond
- magnetic field $\approx 3\ \mathrm{mT}$ for additional splitting
- quantum frequency converter: $637\ \mathrm{nm} \rightarrow 1588\ \mathrm{nm}$
- midpoint: beam splitter and SNSPD



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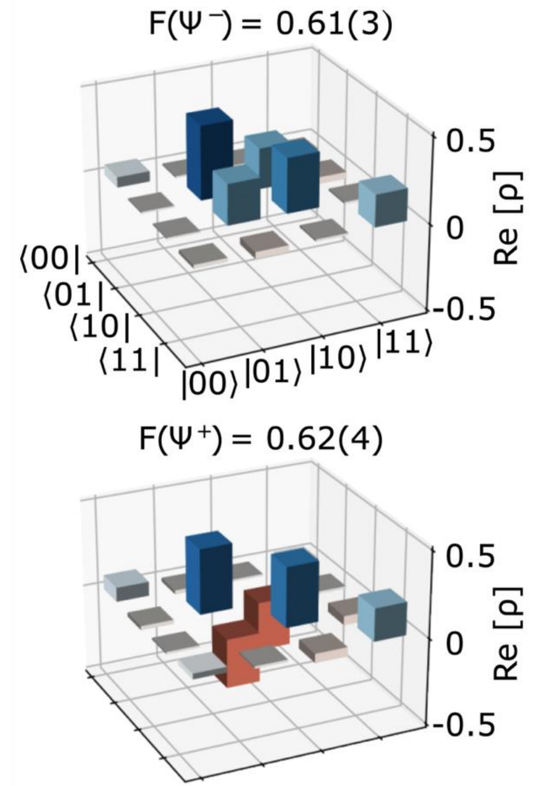
Metropolitan-Scale Entanglement

- postselected entanglement
  - 540 rounds of entanglement
  - $F(\psi^-) = 0.61$ and $F(\psi^+) = 0.62$
  - entanglement generation rate: $0.48\ \mathrm{Hz}$
  - seccess probability: $7.1\ \times 10^{-6}$
- fully heralded entanglement
  - refocusing echo pulse
  - travel time: $52\ \mu s$ and $73\ \mu s$
  - $F(\psi^-) = 0.534$
  - entanglement generation rate: $0.022\ \mathrm{Hz}$



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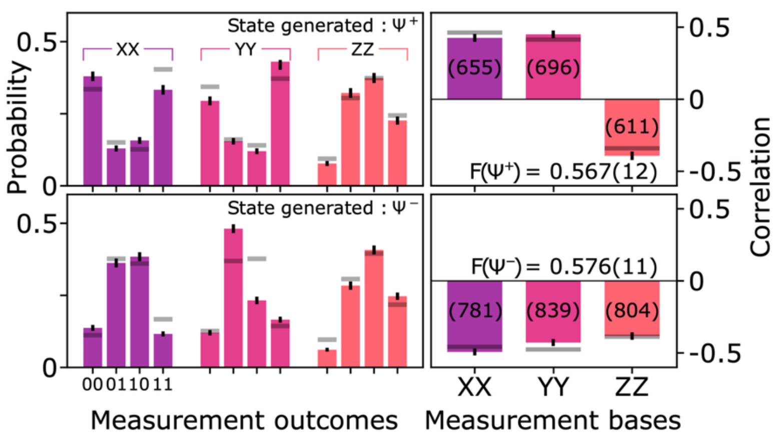

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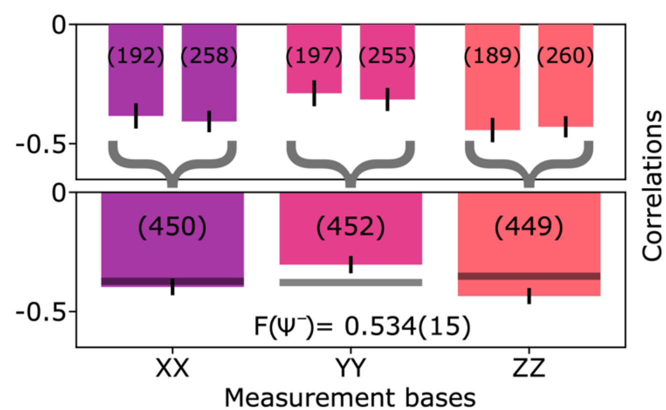

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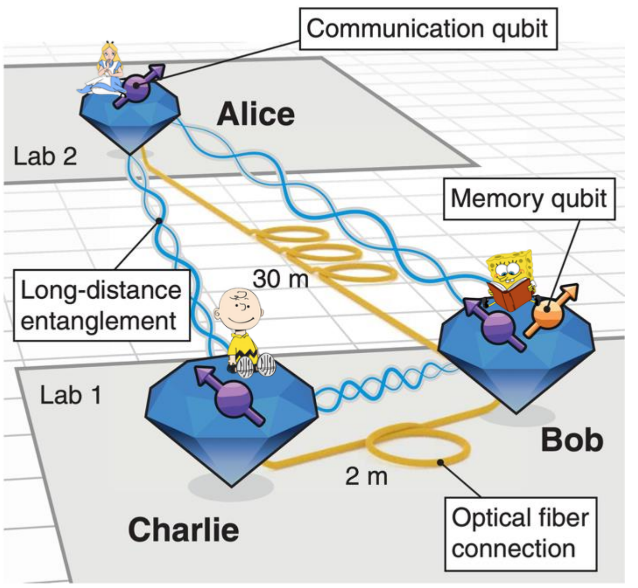
Multinode Quantum Network

- communication qubit
- additional memory qubit
- two key quantum network protocols
- Delft University of Technology, Netherlands
- 3 quantum nodes
- two interferometers
- NV center electronic spin
- additional memory qubit: carbon-13 nuclear spin
- additional magnetic field $\approx 189\,\mathrm{mT}$



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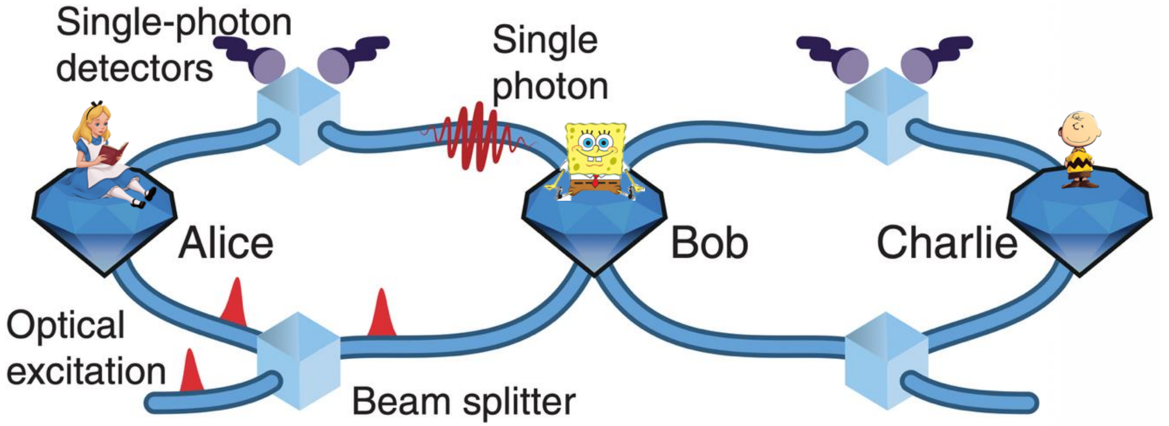

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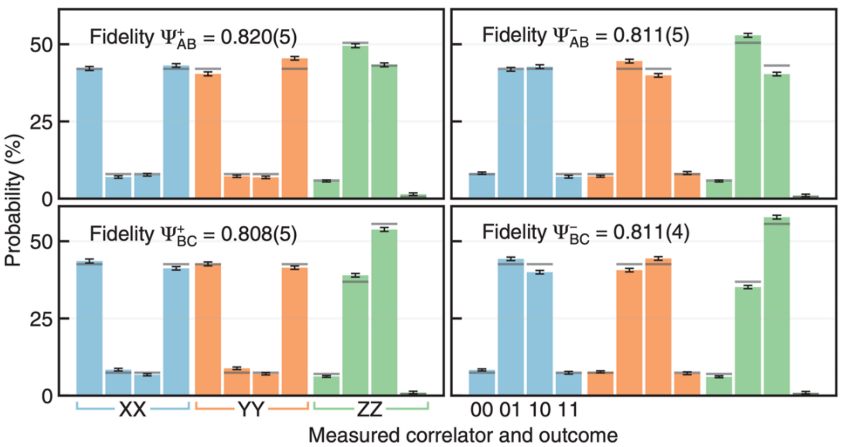
Multinode Quantum Network

- $F(\psi^{\pm}) = 0.8$
- entanglement generation rate: $9\ \mathrm{Hz}$ and $7\ \mathrm{Hz}$
- sequence depicted using circuit diagram
- swap: microwave and radio frequency pulses
- test for success
- two protocols



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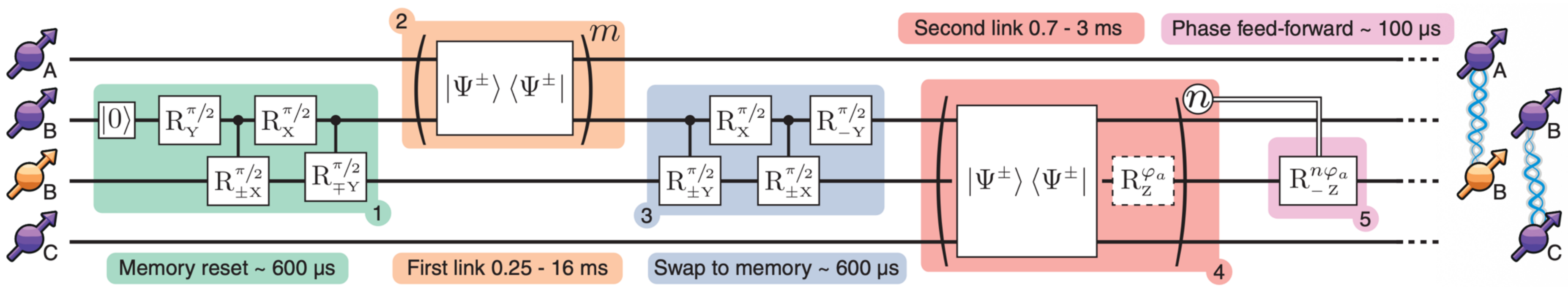

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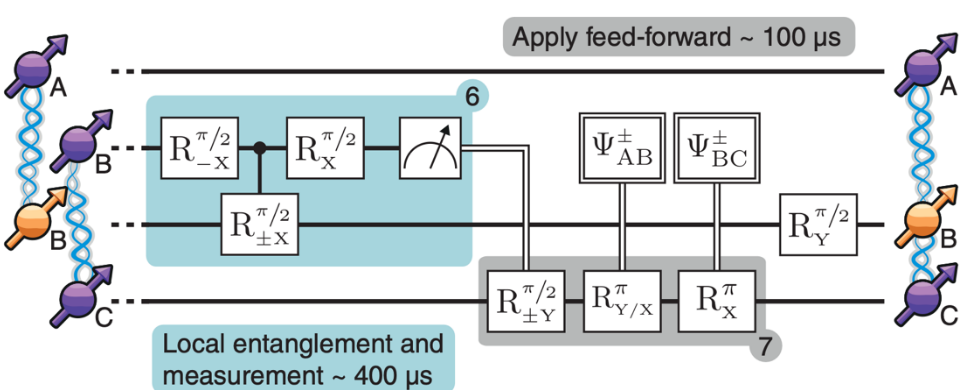
Multinode Quantum Network

1) Distribution of entanglement

- generation of multipartite entangled GHZ state:

$$|GHZ\rangle_{ABC} = \frac{1}{\sqrt{2}}(|000\rangle \pm |111\rangle)$$

- CNOT and Hadamard gate
- $F(GHZ) = 0.538$
- entanglement generation rate: $0.011\ \text{Hz}$



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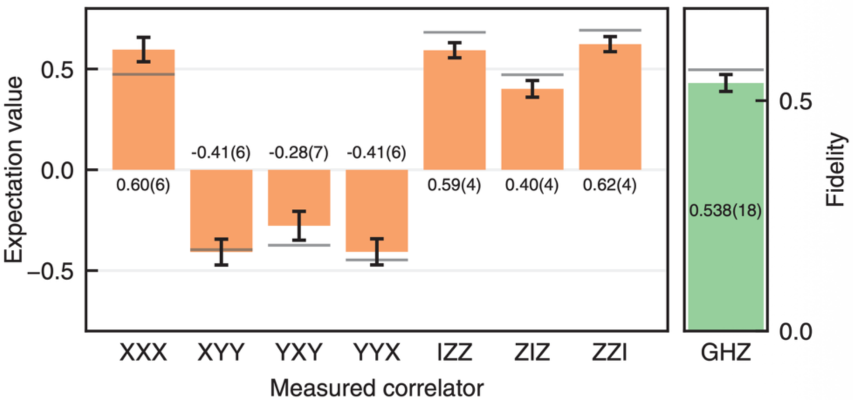

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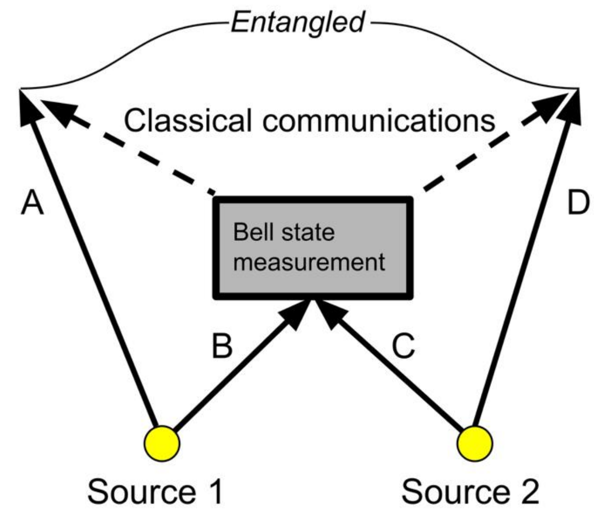
Multinode Quantum Network

2) Entanglement swapping

- Bell state measurement
  $\Longrightarrow$ collaps
- entanglement without direct interaction:

$$|\phi^{\pm}\rangle_{AC} = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$$

- $F(\phi^{\pm}) = 0.551$
- entanglement generation rate: $0.025\ \mathrm{Hz}$



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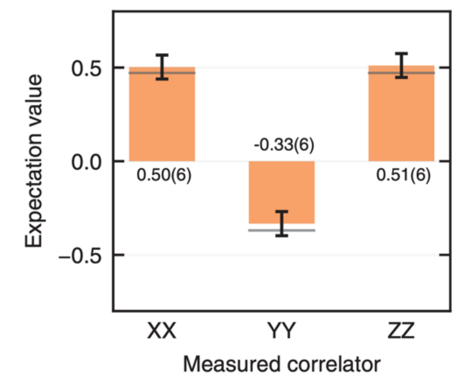

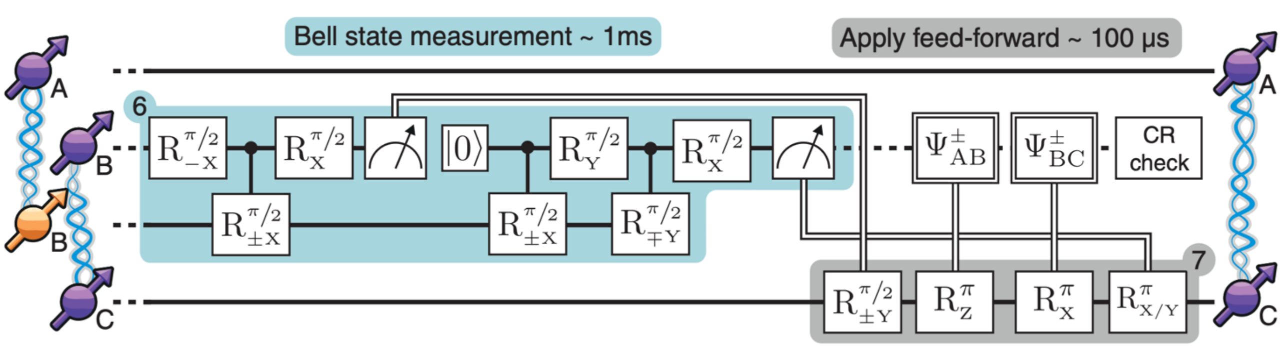

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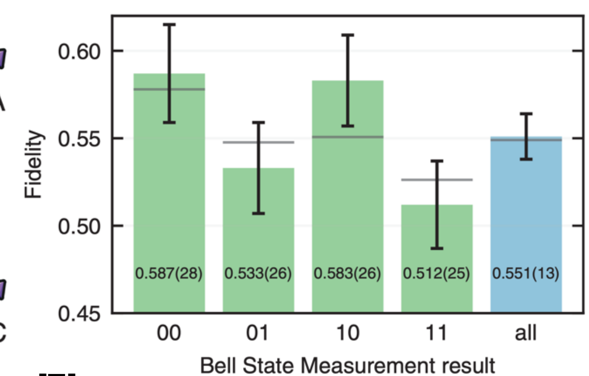

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Conclusion

- connects two particles to each other so that they become inseparable
- NV centers can generate heralded entanglement
- basis for the realization of a quantum network that would offer fast and secure information transfer
- looked at:
  - architecture addresing key scalling challenges
  - platform for exploring, testing, and developing multinode quantum networks

  $\Longrightarrow$ potential of NV-center-based platforms for scalable quantum networks

- future quantum network applications that quantum non-locality to solve problems

[10]

Sources of Figures

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