UNIVERSITY OF LJUBLJANA | FMF Faculty of Mathematics and Physics

Generating Entangled Photon Pairs Using Nonlinear Crystal With Type-II SPDC

Seminar I - 1st year, 2nd cycle
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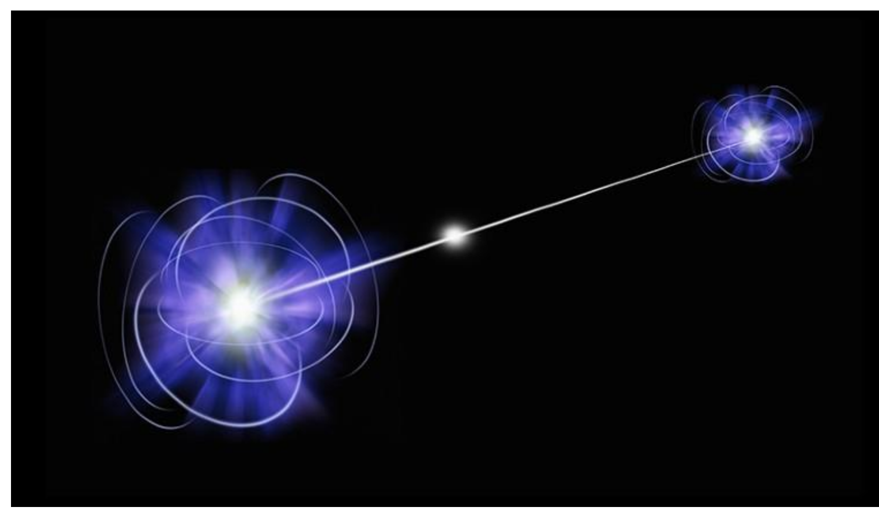
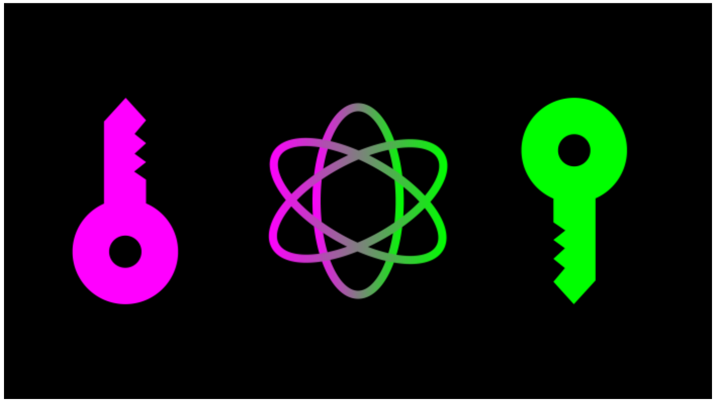
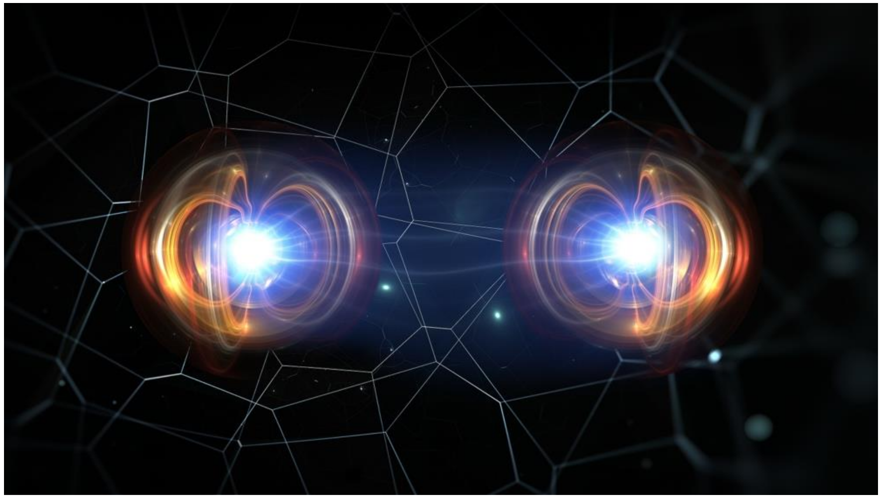


Introduction

- reached field of application
- promise:
  - more precise measurements
  - secure communication
  - implementation of otherwise overly complex simulations
- scalling to larger systems
- using photon polarization
- connect two stationary nodes





Quantum Network

- separate nodes with its own quantum system
- send light through optical fibers
  - state encoded in polarization, frequency or phase
  - fast – speed of light
  - low losses
  - must be isolated
- secure data transmission
- cannot amplify
- limited to a few 100 km
- entanglement swapping
- using satellites

Entanglement

- unit is a qubit
  - any general two-level quantum system
- single qubit state: $|\psi\rangle = \sum_i c_i |\phi_i\rangle$
- two-qubit state: $|\psi_{AB}\rangle = |\psi\rangle_A \otimes |\psi\rangle_B$
- photons ⟶ polarization
  - $|\psi\rangle = \alpha|H\rangle + \beta e^{i\phi}|V\rangle$
- product state:

$$|\psi\rangle = |\psi\rangle_1 \otimes |\psi\rangle_2 = \left(\alpha_1|H\rangle_1 + \beta_1 e^{i\phi_1}|V\rangle_1\right) \otimes \left(\alpha_2|H\rangle_2 + \beta_2 e^{i\phi_2}|V\rangle_2\right) =$$

$$= c_1\,|H\rangle_1|H\rangle_2 + c_2\,|H\rangle_1|V\rangle_2 + c_3\,|V\rangle_1|H\rangle_2 + c_4\,|V\rangle_1|V\rangle_2$$

- separable states
- statistical mixture of states

⇒ density matrix formalism

Entanglement

- density operator: $\hat{\rho} = \sum_i p_i |\psi_i\rangle\langle\psi_i|$
  - pure states: $\hat{\rho} = |\psi\rangle\langle\psi|$
  - mixed states: $\hat{\rho} = \sum_i p_i |\psi_i\rangle\langle\psi_i|$
- density matrix:

$$\rho = \begin{array}{c} \\ |HH\rangle \\ |HV\rangle \\ |VH\rangle \\ |VV\rangle \end{array} \begin{array}{c} \begin{array}{cccc} \langle HH| & \langle HV| & \langle VH| & \langle VV| \end{array} \\ \begin{pmatrix} A_1 & B_1 e^{i\phi_1} & B_2 e^{i\phi_2} & B_3 e^{i\phi_2} \\ B_1 e^{i\phi_1} & A_2 & B_4 e^{i\phi_4} & B_5 e^{i\phi_5} \\ B_2 e^{i\phi_2} & B_4 e^{i\phi_4} & A_3 & B_6 e^{i\phi_6} \\ B_3 e^{i\phi_3} & B_5 e^{i\phi_5} & B_6 e^{i\phi_6} & A_4 \end{pmatrix} \end{array}$$

Entanglement

A state is entangled if it is not separable, meaning we cannot write it as a product state.

- measurements are correlated
- examples:

  1) four *Bell states*

$$|\psi^{\pm}\rangle = \frac{1}{\sqrt{2}}\left(|H\rangle_1|V\rangle_2 \pm |V\rangle_1|H\rangle_2\right)$$

$$|\phi^{\pm}\rangle = \frac{1}{\sqrt{2}}\left(|H\rangle_1|H\rangle_2 \pm |V\rangle_1|V\rangle_2\right)$$

  2) *GHZ state*

$$|GHZ\rangle = \frac{1}{\sqrt{2}}\left(|H\rangle^{\otimes n} \pm |V\rangle^{\otimes n}\right)$$

Entanglement – Characterization

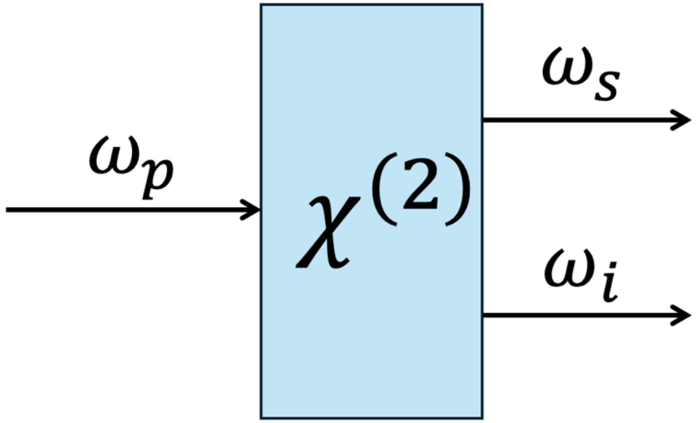
- quality of the entangled state
- fidelity: how close the real state is to the targeted state
  - $F \sim \langle \rho_1 | \rho_2 \rangle$
  - $0 \leq F(\rho_1, \rho_2) \leq 1$
- purity: how much the state is mixed
  - $\gamma = Tr\{\rho^2\}$
  - $0 \leq \gamma(\rho) \leq 1$
- tangle: measure of entanglement
  - $0 \leq T(\rho) \leq 1$

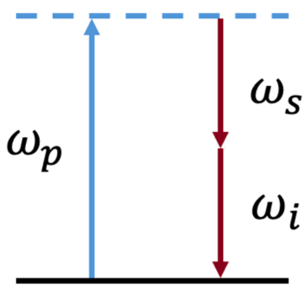
Spontaneous Parametric Downconversion – SPDC

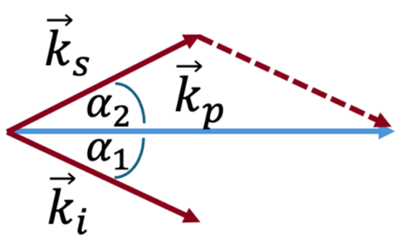
- second-order nonlinear optical effect
- strictly quantum effect
- splits into two photons
- energy conservation
  - $\omega_p = \omega_s + \omega_i$
- momentum conservation
  - $\Delta\vec{k} = \vec{k}_p - \vec{k}_s - \vec{k}_i$

  $\Rightarrow$ phase matching conditions
- inefficient process

  $\Rightarrow$ benefit

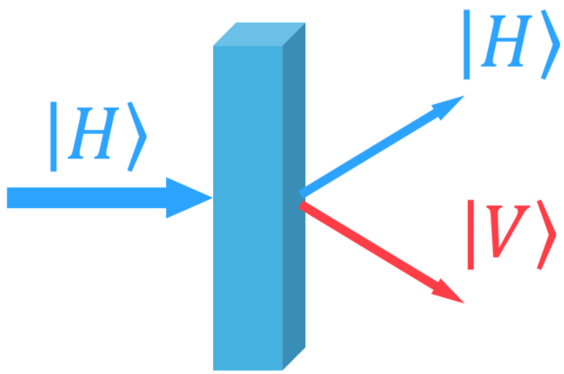




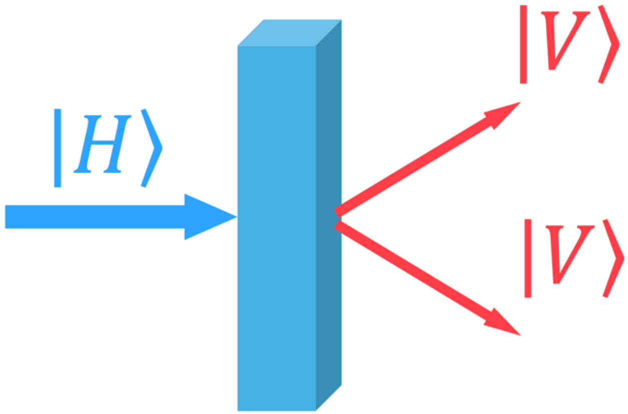


SPDC – Types

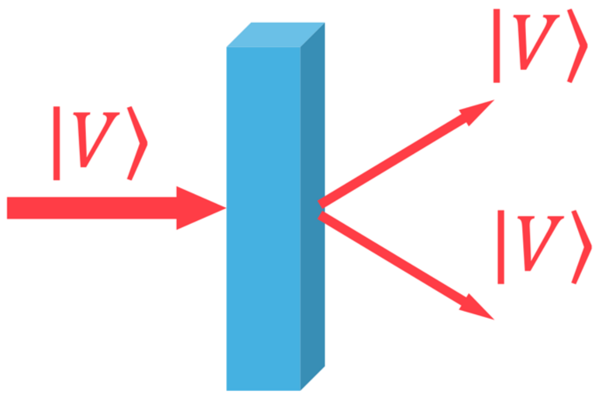
- type-II



- type-I



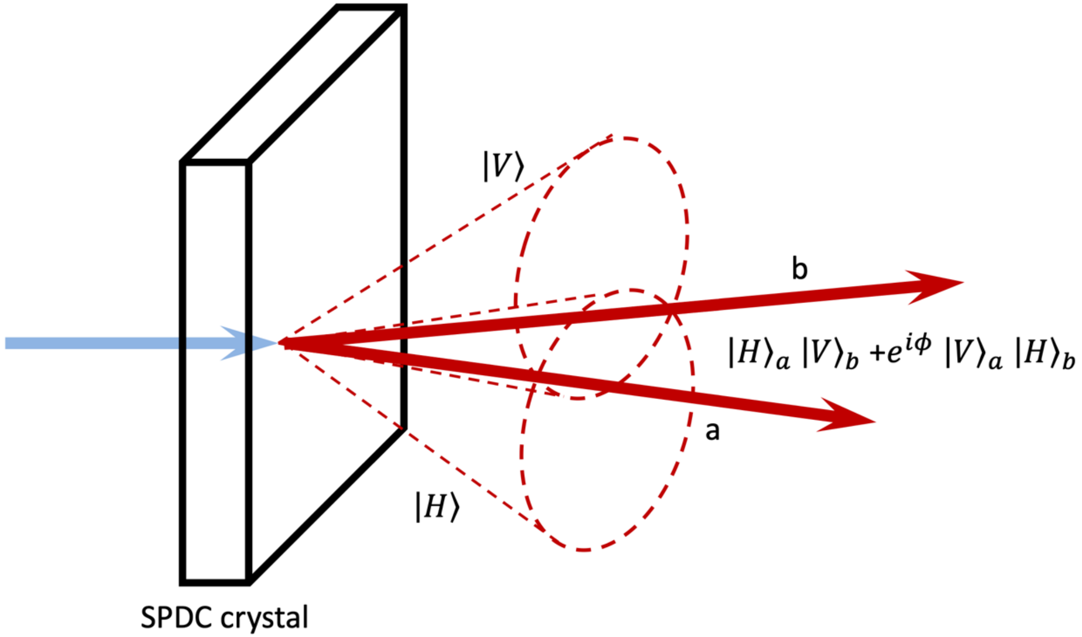
- type-0



SPDC – Nonlinear Crystals

$$k_p = k_s + k_i \xrightarrow{k_j = \frac{n_j \omega_j}{c}} \frac{n_p \omega_p}{c} = \frac{n_s \omega_s}{c} + \frac{n_i \omega_i}{c}$$

- normal dispersion: $n_i(\omega_i) < n_s(\omega_s) < n_p(\omega_p)$ for $\omega_i < \omega_s < \omega_p$
  - ⇒ birefringent crystals
- refractive indices: n_e and n_o
- birefringent phase matching
  - $\Delta k = 0$
- cone-like distribution



SPDC – Walk-off

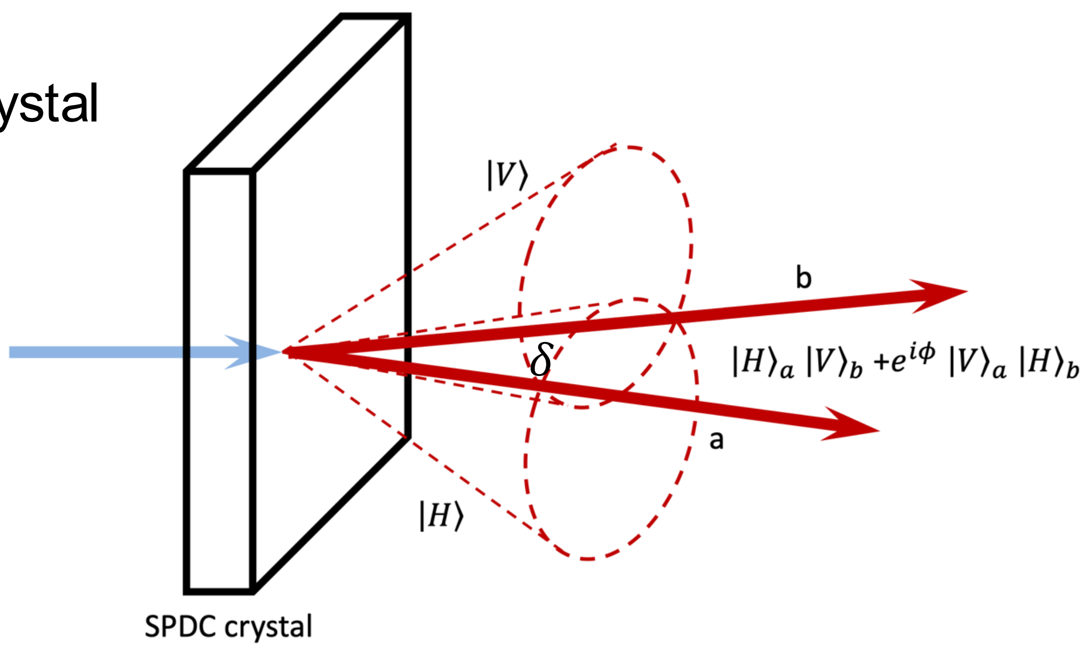
- input beam and generated beams do not travell in the same direction
- temporal walk-off:

$$\Delta t = \left| L_c \left(\frac{1}{n_e} - \frac{1}{n_o} \right) \right|$$

- spatial walk-off:

$$\delta = -\frac{1}{n_e} \frac{dn_e}{d\theta}$$

- limits the length of the crystal



SPDC – Quasi-phase Matching

- $\Delta k = k_p - k_s - k_i \to 0$ ... hard to satisfy

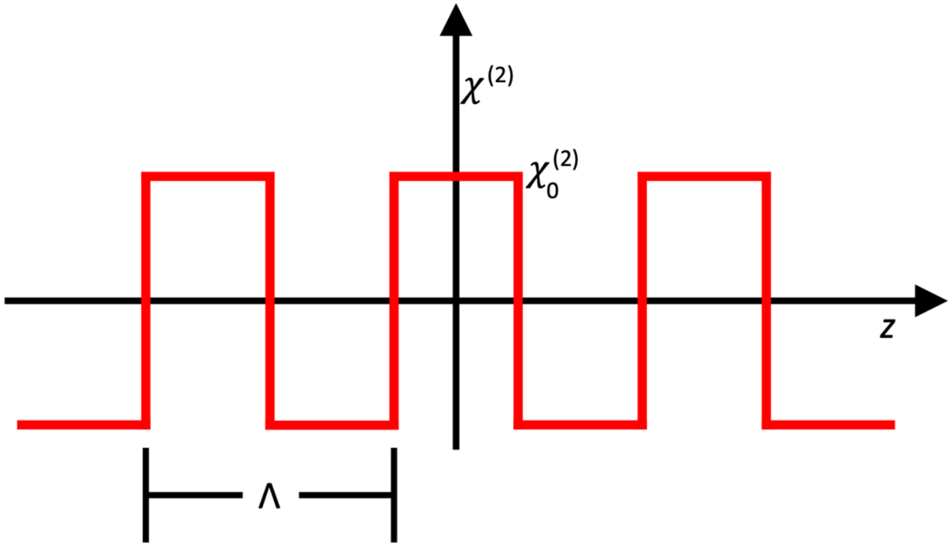
  $\Rightarrow$ modify the material

- quasi-phase matching condition:

$$\vec{k}_p = \vec{k}_s + \vec{k}_i + \vec{K}_G \;,\; \text{where} \;\; K_G = \frac{2\pi}{\Lambda}$$

  - $\Lambda : 10 - 100\ \mu m$

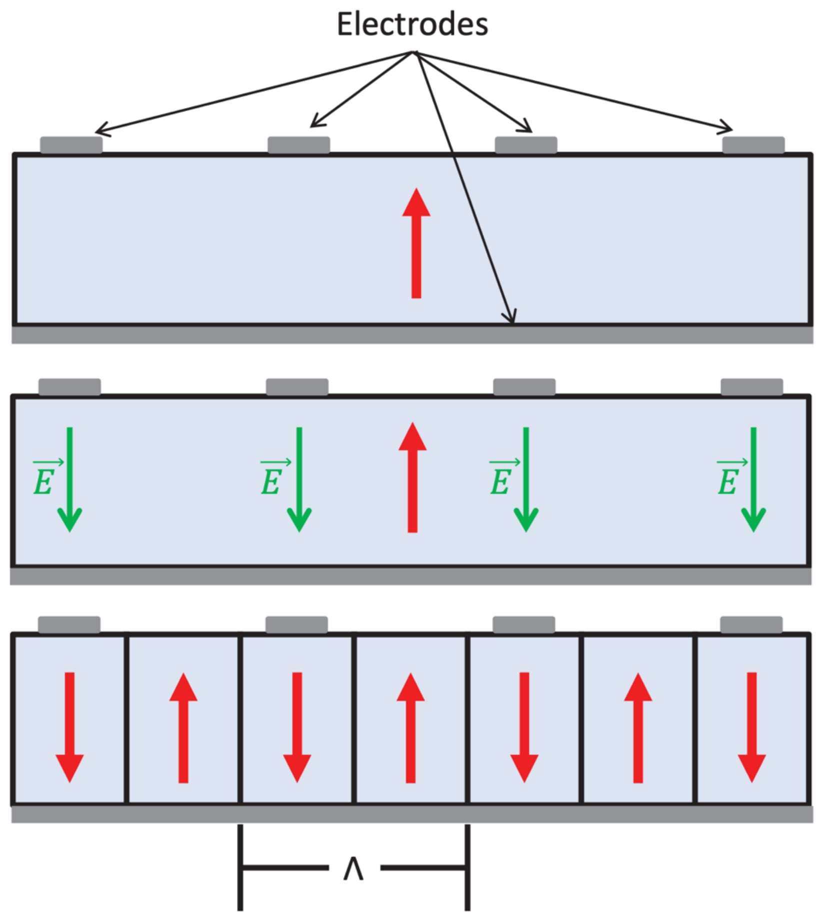
- no angle tuning
  - collinear downconversion
  - travel along crystal axis
- can achieve type-0 SPDC



SPDC – Periodic Poling

- 2 methods:
  1) thin slices, where every other is rotated
  2) periodic poling
- ferroelectric materials
  - LN: lithium niobate
  - LT: lithium tantalate
  - KTP: potassium titanyl phosphate

  linear susceptibility: $\sim 10^{-11}$m/V
- field: $\sim$10 kV/mm
- need temperature tuning
  - $n = n(T)$



SPDC – Periodic Poling

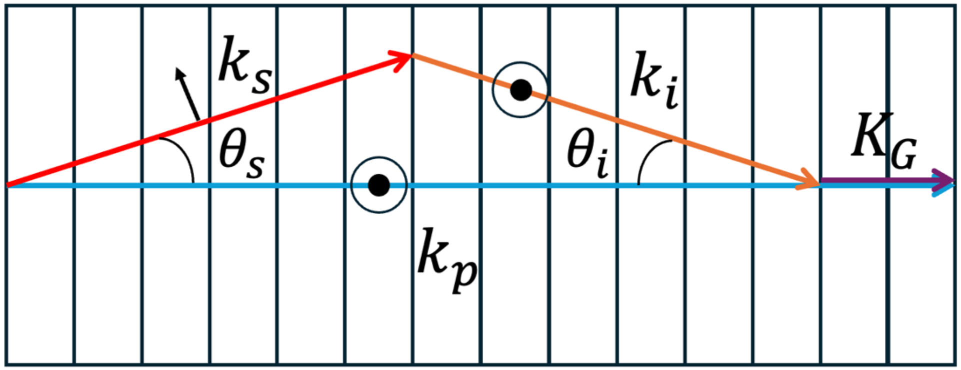
$$\vec{k}_p = \vec{k}_s + \vec{k}_i + \vec{K}_G \xrightarrow{angle} \begin{array}{c} k_s \cos\theta_s + k_i \cos\theta_i = k_p - K_G \\ k_s \sin\theta_s = k_i \sin\theta_i \end{array}$$

- can express: $\vec{k}_j = \frac{2\pi \vec{n}_j}{\lambda_j}$
- collinear type-II SPDC:
  - ellipsoid equation

$$n_s(\theta_s, T, \lambda_s) = \frac{n_x(T,\lambda_s)\, n_z(T,\lambda_s)}{\sqrt{n_x^2(T,\lambda_s)\cos{\theta_s}^2 + n_z^2(T,\lambda_s)\sin{\theta_s}^2}}$$

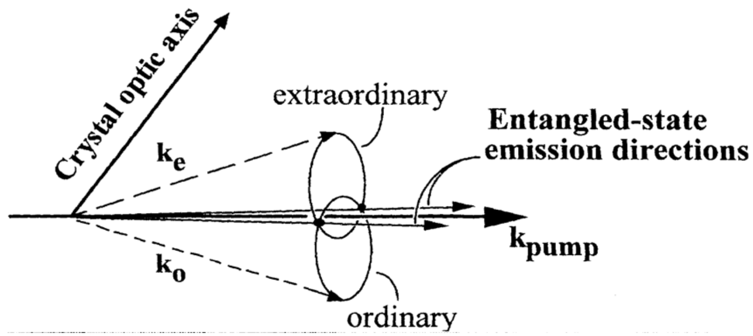
  - Sellmeier equations

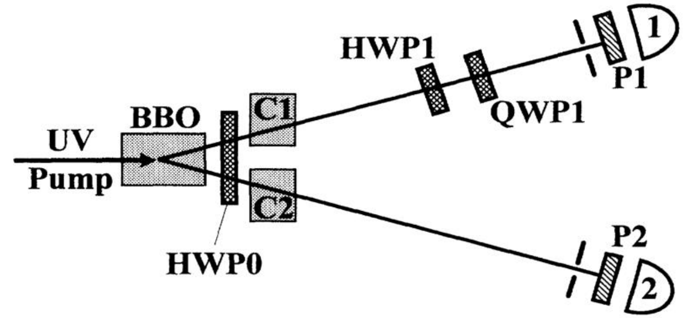
$$n_p = n_y(T, \lambda_p)$$
$$n_i = n_y(T, \lambda_i)$$



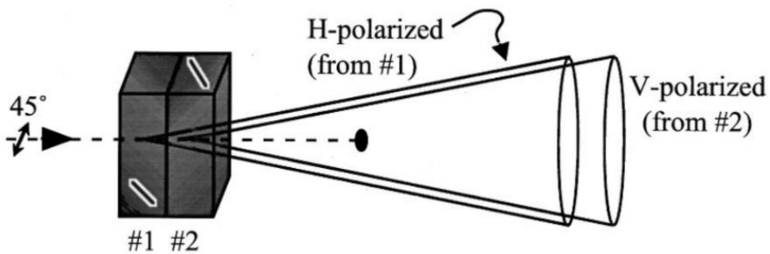
Generation of Polarized Entangled Photon Pairs

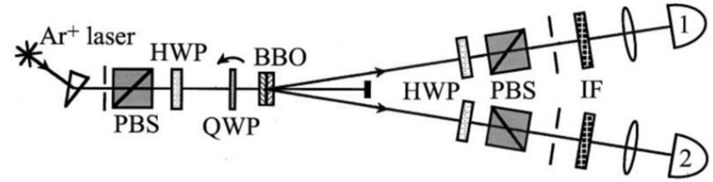
1) single emitter for type-II SPDC





2) sandwich scheme for type-I SPDC





3) interferometric methods

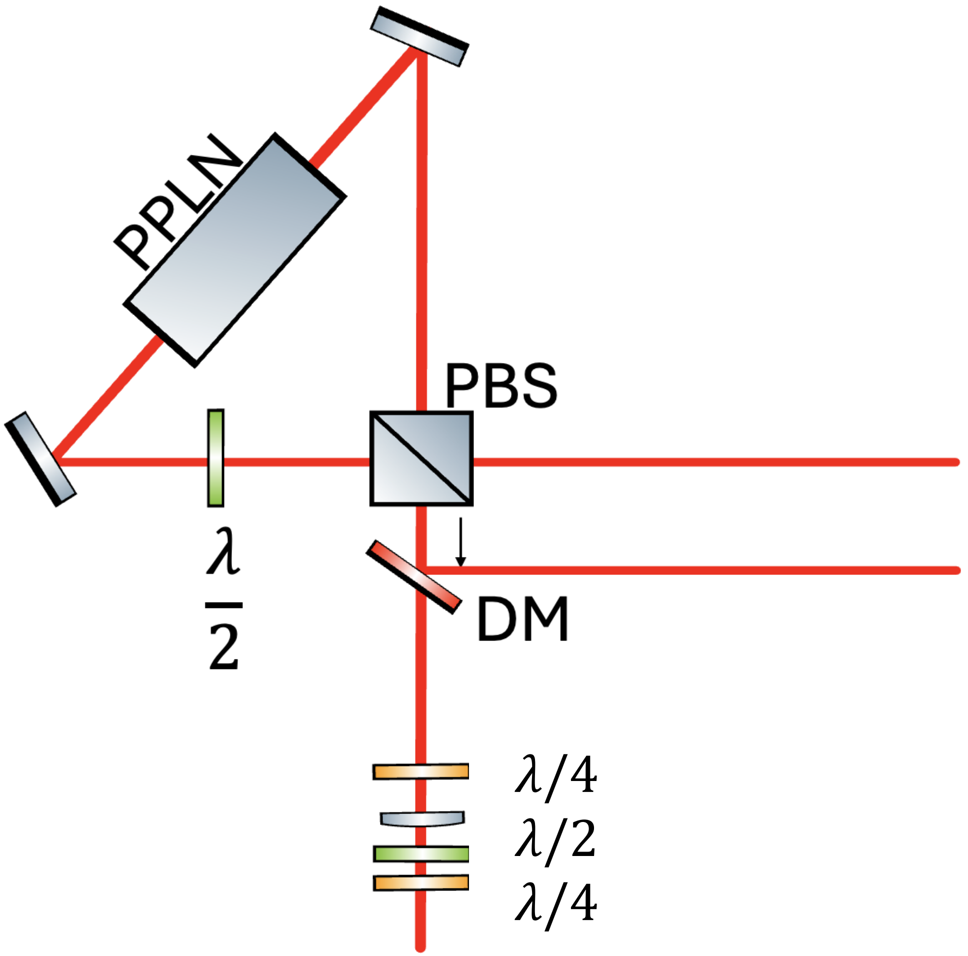
- Mach-Zehnder interferometer
- Sagnac interferometer

Sagnac Interferometer

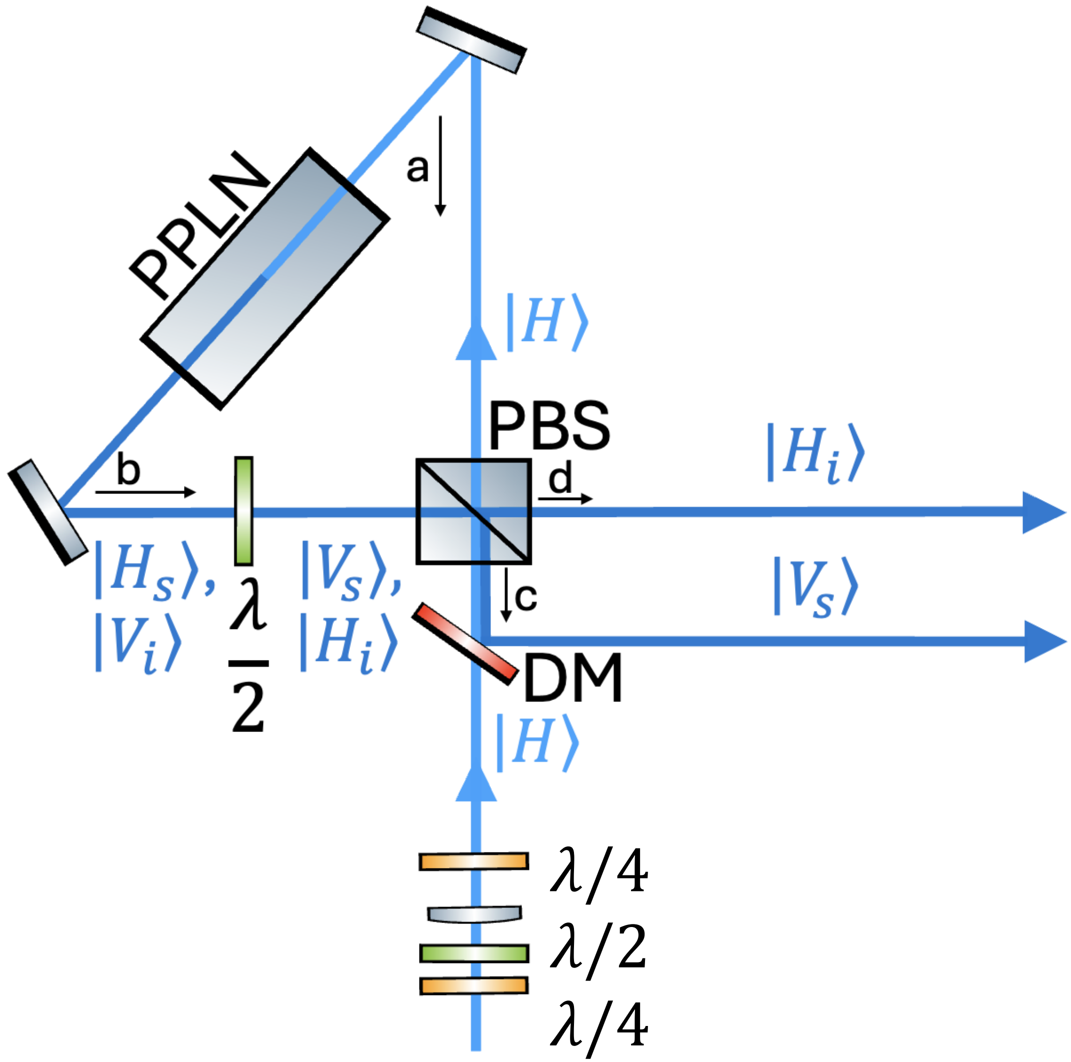
- phase-stable
- QPM: $|H_p\rangle \rightarrow |H_s\rangle, |V_i\rangle$

  ⇒ helps with imperfections of PBS
- pump polarization
- finer tuning
- plano-convex lens
- dichroic mirror
- band pass filter and

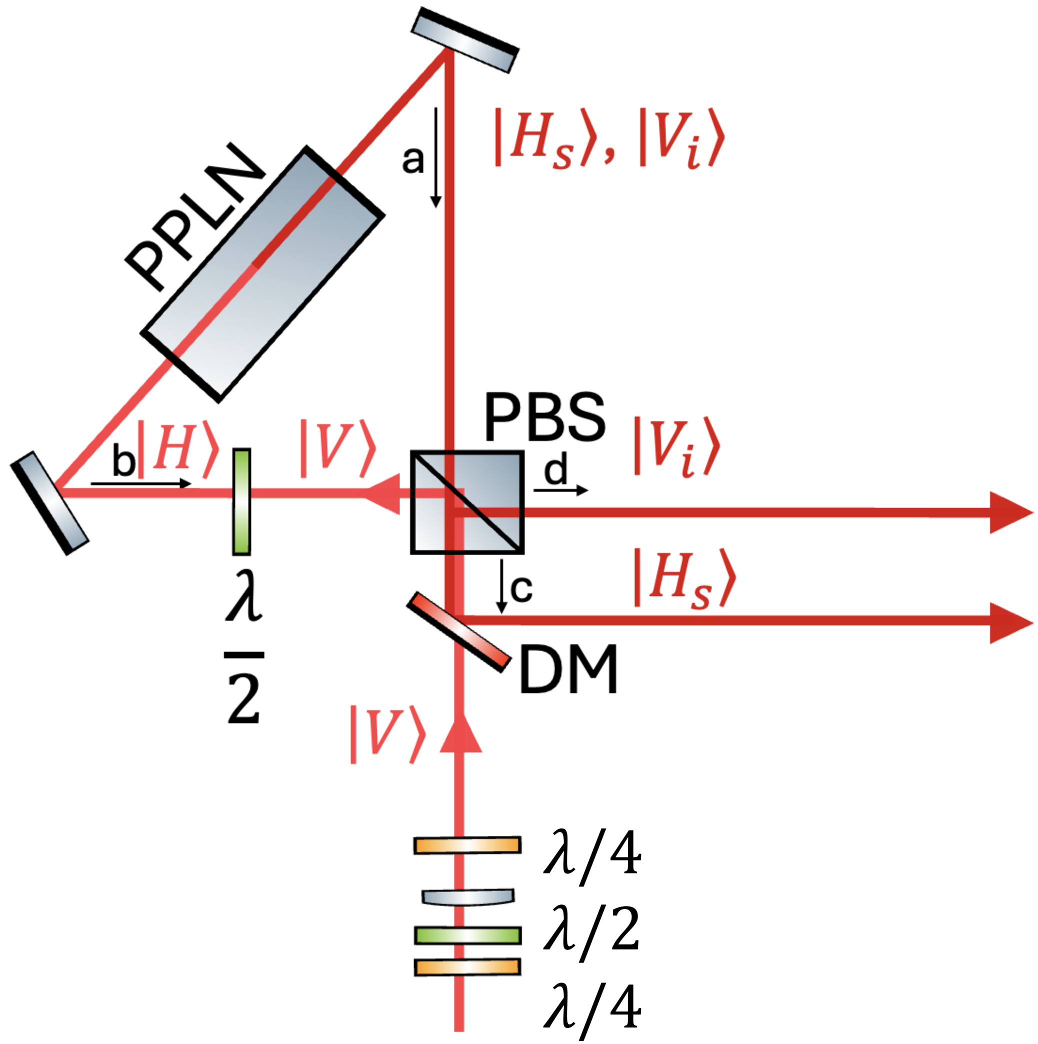
  multiplexer



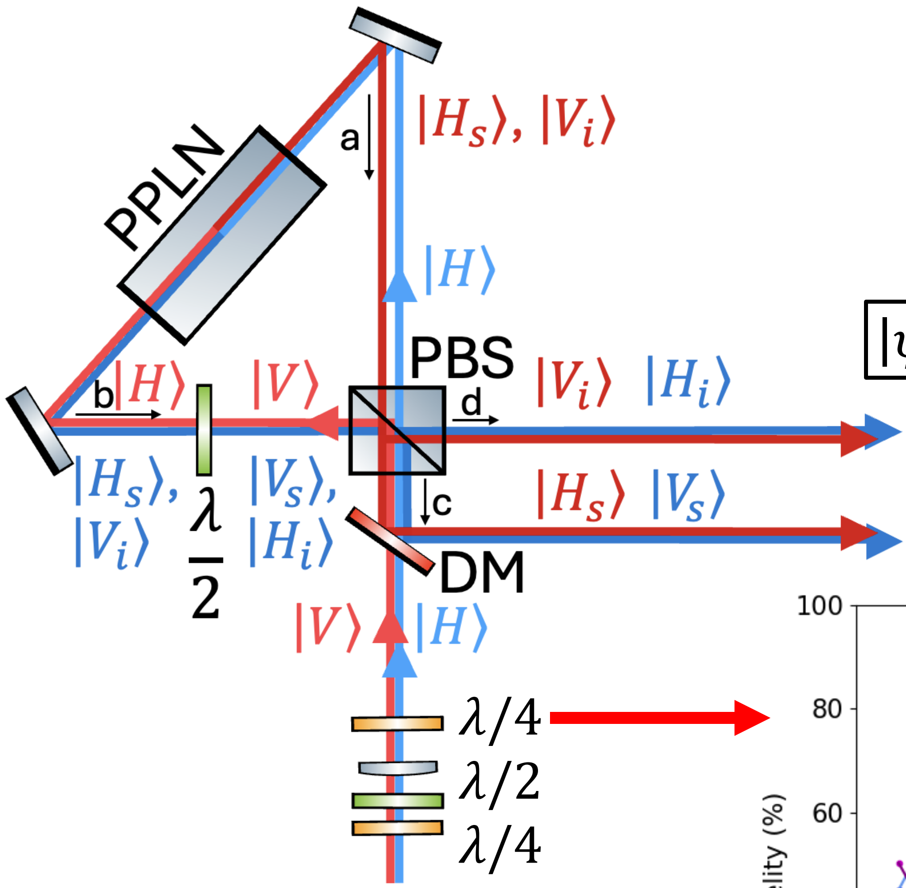
Sagnac Interferometer – loop $|H\rangle$



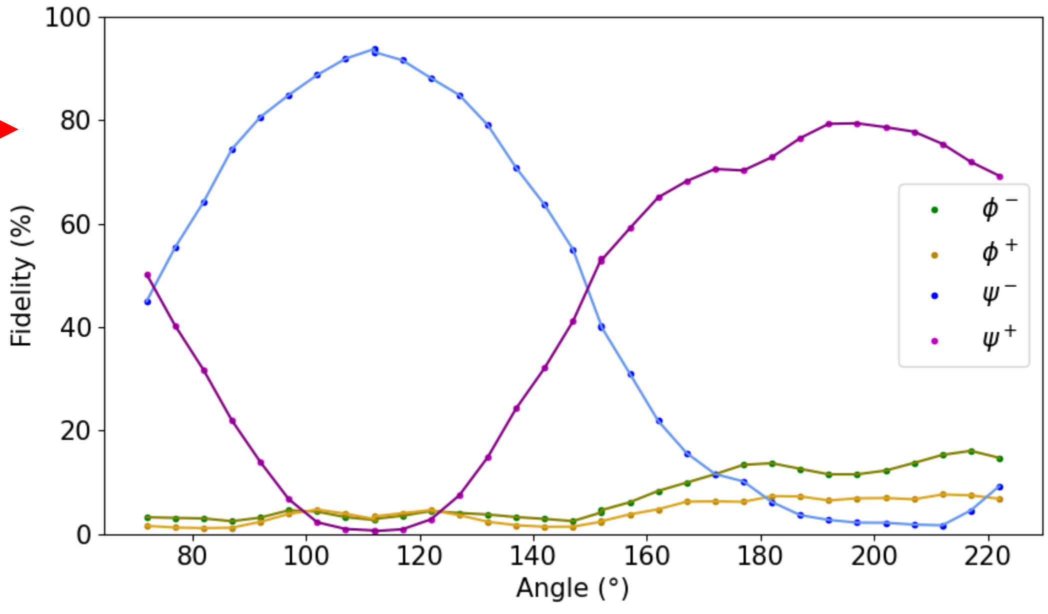
Sagnac Interferometer – loop $|V\rangle$



Sagnac Interferometer

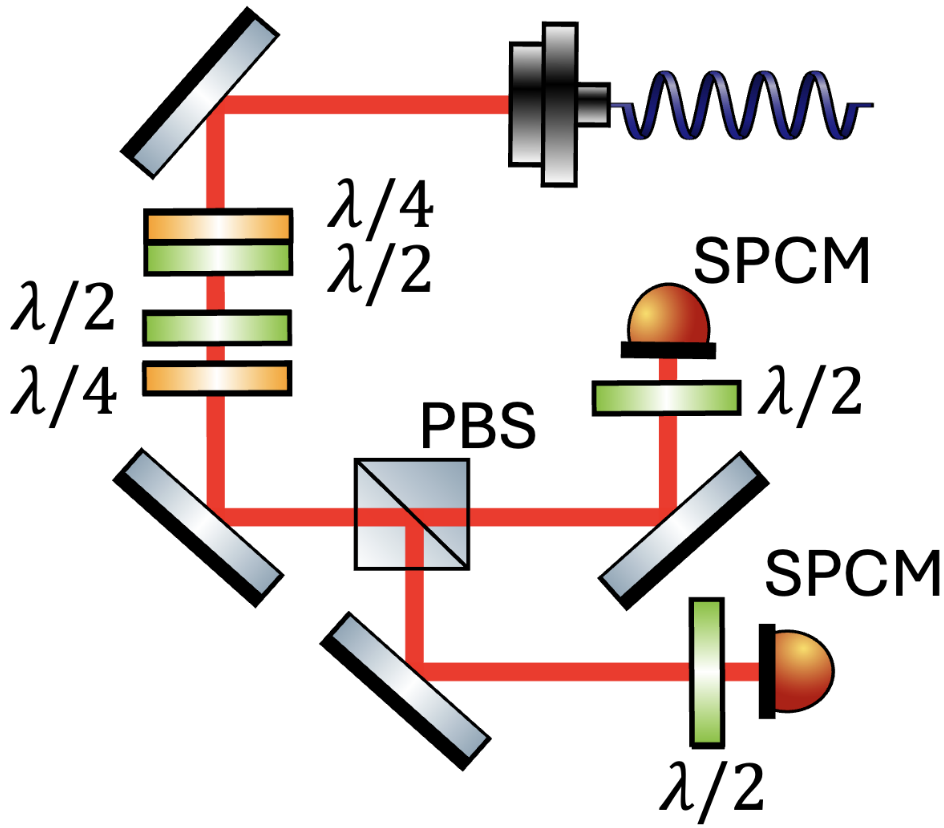


$$|\psi\rangle = |H_s\rangle_c|V_i\rangle_d + \beta e^{i\phi}|V_s\rangle_c|H_i\rangle_d$$



Entanglement Measurement and Its Interpretion

- quantum state tomography
  - maximum likelihood method
- reconstruct the density matrix
- external setup
- compensating wave plates
- choose measurement basis
- project in different bases
  - linear
  - diagonal
  - circular

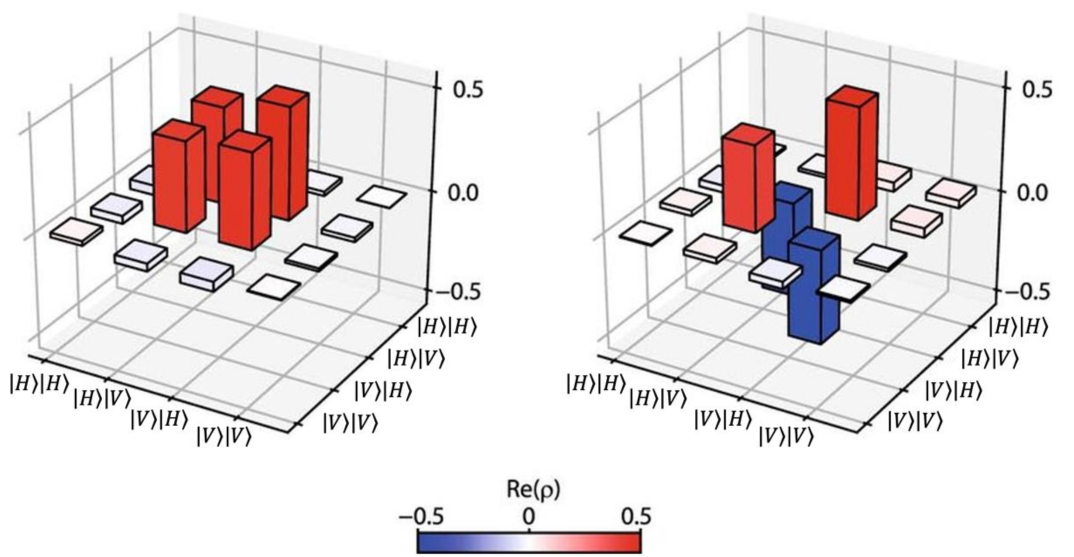


Tomography Results

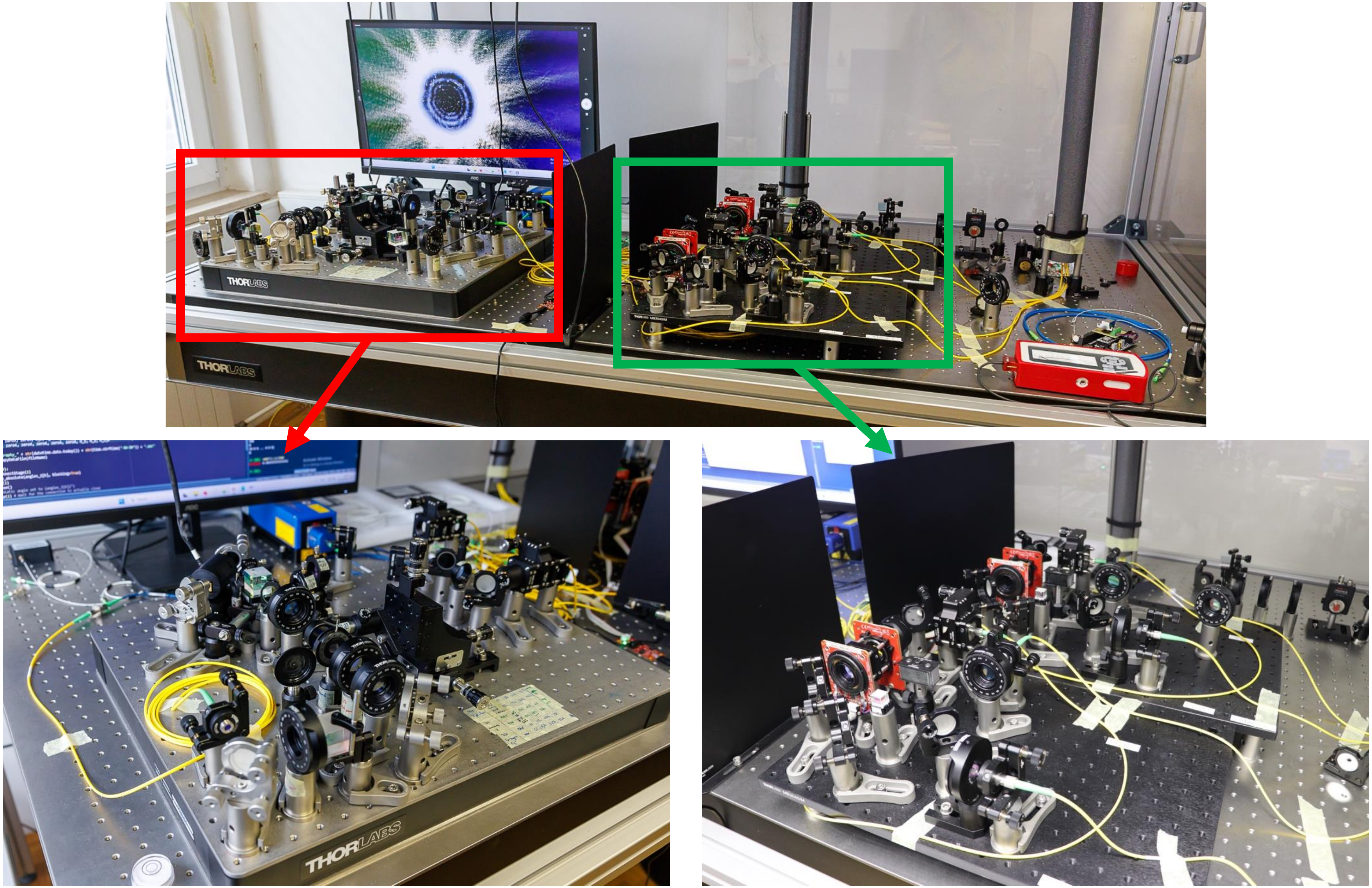
1) density matrix

$$\rho = \begin{array}{c} \\ |HH\rangle \\ |HV\rangle \\ |VH\rangle \\ |VV\rangle \end{array} \begin{array}{c} \begin{array}{cccc} \langle HH| & \langle HV| & \langle VH| & \langle VV| \end{array} \\ \begin{pmatrix} A_1 & B_1e^{i\phi_1} & B_2e^{i\phi_2} & B_3e^{i\phi_2} \\ B_1e^{i\phi_1} & A_2 & B_4e^{i\phi_4} & B_5e^{i\phi_5} \\ B_2e^{i\phi_2} & B_4e^{i\phi_4} & A_3 & B_6e^{i\phi_6} \\ B_3e^{i\phi_3} & B_5e^{i\phi_5} & B_6e^{i\phi_6} & A_4 \end{pmatrix} \end{array}$$

2) histogram for real and imaginary amplitudes

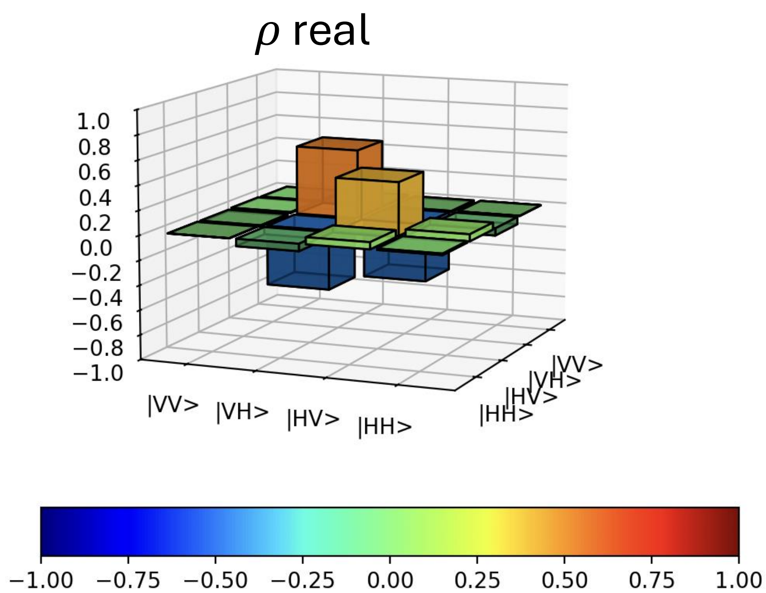


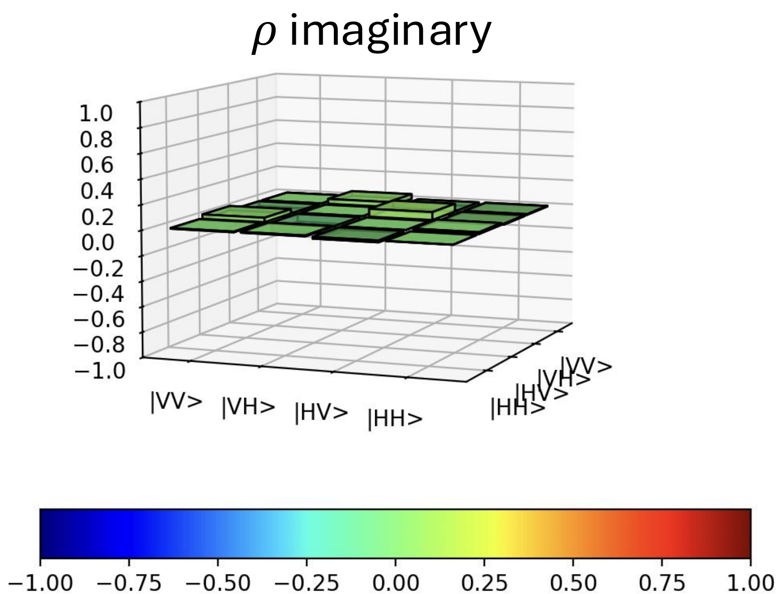
Implementation of Experiment – IJS



Implementation of Experiment – IJS

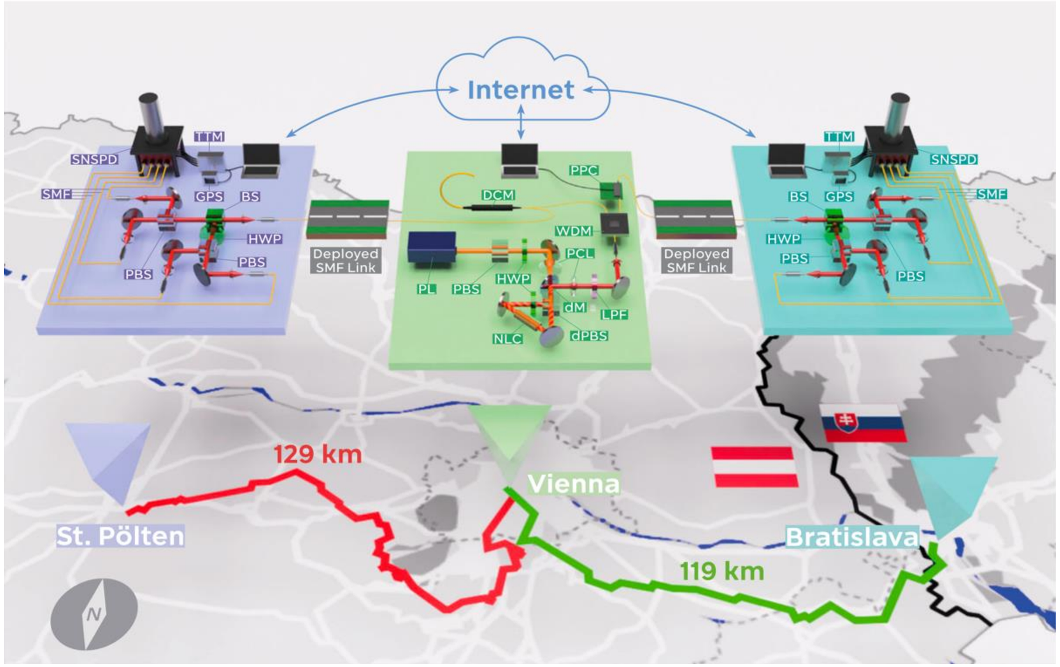
- used:
  - 780 nm pump laser
  - PPLN crystal $(50 \times 2 \times 1$ mm)
  - 1550 nm downconverted photons
  - 250 mm plano-convex lens
- results:
  - fidelity: $(97.1 \ \pm 0.08)$ %
  - purity: $(97.9 \ \pm 0.09)$ %
  - tangle: $(92.5 \ \pm 0.07)$ %





Implementation of Experiment Over Record Long Distance

- Austrian Academy of Science
- over 248 km in optical fibers
- used:
  - 775.06 nm pump laser
  - PPLN crystal
  - 1550.12 nm downconverted photons
- fidelity: $> 99\%$
- independent of weather



Conclusion

- basis for:
  - realization of quantum communication
  - quantum teleportation
- important for quantum key distribution (QKD)
- transfer of information between particles that are far apart
- robust and easy to use method