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Seminar I – 1st year, 2nd cycle

The DLCZ Protocol

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In this seminar, I explain a quantum communications protocol, capable of arbitrarily long connections that scale only polynomially in their preparation time. It is named after Duan, Lukin, Cirac and Zoller. I explain how a spontaneous Raman transition can be used for entanglement creation and swapping of collective atomic states with the help of entanglement via a beam splitter. I will explain how one can realize it, how some of its underlying technologies work and why its understanding is essential in the development of quantum communications.

Contents

1 Introduction

1.1 Overlook

Quantum communications is a quickly developing field, trying to achieve the same as we already have in classical communications, but by employing quantum effects. Namely, the data carriers are quantum objects. With these effects many new and classically impossible schemes are becoming feasible. Such are completely secure quantum key distribution (QKD) as with the BB84 protocol [1] and farther quantum state transmission for quantum networks or for distant entanglement – this is the topic of this seminar. Here, other QKD schemes like the Ekert protocol [2] or quantum state teleportation [3] become especially beneficial. The field's progress follows the development of quantum technologies, like single photon sources, single photon detectors, quantum memories, because these all make harnessing the features of quantum processes more and more simple.

For example, the mentioned complete security is possible due to the physical reality of the quantum world. Let's say, there is an eavesdropper trying to listen to an ongoing connection. They would need to perform quantum measurements on the data carriers. But as encapsulated in the fifth postulate of quantum mechanics, this would affect the data carriers' states. This meddling with the connection can in principle be detected by the channel's legitimate users.

1.2 Difficulties

Transmitting quantum carriers through space, like between satellites [4] or over optical fibers, always leads to exponential decay of the quality of our signal with the path travelled. This is due to absorption of particles and destruction of quantum states. So, we would like to amplify this signal before it dies out beyond the possibility of restoration. This is a common practice in classical communication, but here in quantum mechanics, we have a couple set-backs in comparison.

We might for example think of the process of stimulated emission of light from an excited atom. There, the photon, interacting with the excited atom, induces the transition of the atom into a lower energy state, where a photon with exactly the same frequency, phase, direction, polarization is emitted in the process (of course if such transition can and does actually happen). By the looks of things, we have just *cloned* the photon. And this is perfectly possible, if its polarization was already known beforehand. But it is impossible to clone an arbitrary state [5]. Such an operation would break the linearity of quantum mechanics. This is also famously called the no-cloning theorem.

Another set-back is the quantum decoherence. This is the process of a pure quantum system losing some of its quantum properties and decaying into a classical mixture of states – it is the decay of coherence. A pure quantum system is one about which we know everything we want to or can. This might sound like the first postulate of quantum mechanics, "The state of a system is described by the wave function $\psi(x,t)$" [6], because it is. This could be a system of atoms, all prepared in spin up. If we are interested in the system's spin state, we are already done; it is up. This is a pure state. In this same system we are not interested, for example, also in the positions of atoms and so we do not care if we know nothing about the position state; we know everything we want. On the contrary, if a state is not pure, it is mixed. This means our system has some state probability distribution and so we can expect its states only with these probabilities. A mathematical object for the description of more complex quantum systems is the density matrix ρ. It can describe pure quantum states as well as mixed states. Its form for a pure state $|\psi\rangle$ is $\rho = |\psi\rangle\langle\psi|$. Here we have written the state in the "bra-ket" notation and the density matrix as an outer product of this state on itself. We can get a quick picture of decoherence [7], if we look at some pure state, let's say $|\psi\rangle = \frac{1}{\sqrt{2}}(|a\rangle + e^{i\theta}|b\rangle)$, where $|a\rangle$ and $|b\rangle$ are this system's eigenstates. Here, I gave only the second state a phase factor, since we are only interested in the local phase, whereas the global phase is physically unimportant (global gauge transformation invariant). This state's density matrix is then

$$\rho = \frac{1}{2}\begin{bmatrix} 1 & e^{-i\theta} \\ e^{i\theta} & 1 \end{bmatrix}.$$

If we imagine this describes a system of multiple subsystems (e.g. particles) prepared in the state $|\psi\rangle$, these subsystems inevitably interact with each other and the environment. The phase factor from each of these pure states consists a unit vector, a vector that lies somewhere on a unit sphere. If in these interactions the subsystems' phases are randomly changed, the average of the phase factors now lies within the unit circle. This means after some time, this average will come to zero. And so the off-diagonal matrix elements vanish, and we are left with just the two diagonal 1/2 matrix elements. This means that we now have a classical mixture of states $|a\rangle$ and $|b\rangle$, where the classical probability of measuring either of the states is 1/2

$$\rho = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix}.$$

Whenever our system is coupled with the outside world (and to some degree it always is), we have decoherence. This will essentially be our culprit in this seminar.

1.3 Way forward

Evidently, transmitting quantum sates through space has some upper limitations on distance, but we can help ourselves to connect smaller distances with entanglement into more distant connections.

The DLCZ protocol consists of implementing pair-wise entanglement between remote links of a chain as A∼B, C∼D, ..., W∼X, Y∼Z. We then connect neighboring links to one another, again by entanglement, as B∼C, D∼E, ..., X∼Y. When the whole chain is connected via entanglement between neighbors, so are the first and the last link of the chain as well, as seen in Fig. 1.

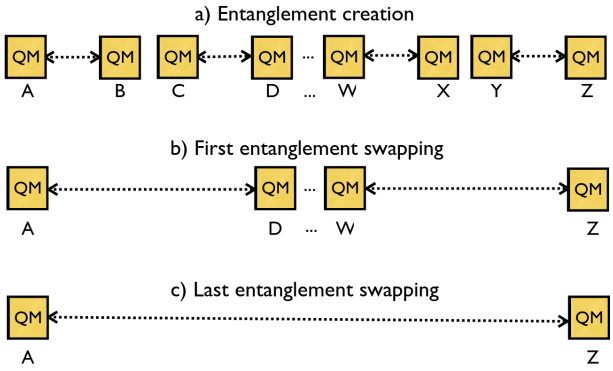

Figure 1: The scheme of the DLCZ protocol [8].

Spatial separation between links leads do decoherence, so the transmitted pure states are received as mixed states. We first need to purify the shared states, so that we can reliably entangle the links. Since not every attempt at entanglement of link pairs will be successful, we need to have some spare time, when states do not change, to retry it until we succeed. If during this time the state at some link would decohere, we obviously would not be able to entangle the whole chain, but would have to prepare that link again. Without this spare time it would be exponentially unlikely to connect all the links at the same time. Therefore, at each point in our chain we need a sort of memory, that can hold such state until all other links are entangled as well. For this a type of quantum memory is used.

Hence, in this seminar I will explain what quantum memories are and what properties they need for the DLCZ protocol, how we generate entangled states between them and how all this comes practically together to create long-distance quantum communication.

A heads-up: I will be jumping between the first (simple, not many-body quantum mechanics, numbers in bras and kets represent states) and second quantization (many-body quantum mechanics, numbers in bras and kets represent numbers of objects in these states) quite a bit. But this destinction should be obvious from the context.

2 Quantum Memory

The idea of quantum memory is to somehow store quantum pure or mixed states for some duration. This means we somehow write some information onto it and after some time, we read it out. We want the output to

- be in the same state, which is described by fidelity (if we write on it the state $|\psi(t=0)\rangle$ for the density matrix $\rho(t=0) = |\psi(t=0)\rangle\langle\psi(t=0)|$, what is the overlap of $\langle\psi(t=0)|\rho(t=\tau)|\psi(t=0)\rangle$ at some later time $\tau > 0$),
- be efficient (what is the ratio of the written and read-out amount of data carriers; for light this is energy ratio),
- have long storage time (for how long we can store this data and whether we can read it out in continuous intervals or at discrete times).

2.1 Types of Quantum Memory

Types of quantum memory [9] are solid state defects (nitrogen vacancy defects in diamondsm, quantum dots in semiconductors), ions (trapped ions, rare-earth ions) or atoms, either on single atoms or atomic ensembles. The latter ones we can either work with in the cold regime [10], or on their vapors. One of the main benefits of working with atomic vapors is its practicality and quick deployability, since no elaborate cooling mechanisms are needed for it. On the other hand, dealing with hot gas leads to higher Doppler shifts and so accessibility to a smaller portion of atoms, population escape from our laser beams because of more rapid movement, more mixing of different atom populations and so on.

2.2 Quantum Memory in Atomic Ensembles

Normally, alkali atoms are used for atomic memories, because of their simplicity (only one valent electron) and so for their "predictability" (very similar to the well studied hydrogen atom). On these, we usually employ the D_1 or D_2 transitions in their hyperfine energy splitting. This splitting's underlying hyperfine structure is a result of the coupling of the total electron angular momentum $\vec{J} = \vec{S} + \vec{L}$ with the total nuclear angular momentum $\vec{I}$. Here, $\vec{L}$ is the orbital angular momentum of the outer electron and $\vec{S}$ is its spin. So, the total hyperfine atomic angular momentum is $\vec{F} = \vec{J} + \vec{I}$. The range of J and F is $|L-S| \leq J \leq L+S$ and $|J-I| \leq F \leq J+I$, respectively.

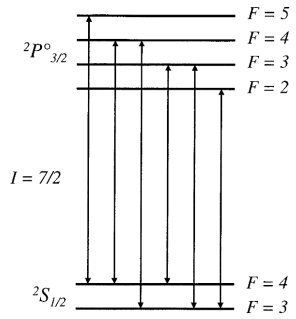

Figure 2: Hyperfine structure with allowed transitions of the ^{133}Cs ($I = 7/2$) D_2 line [11].

If we take a look at $L = 0$, since for electron $S = 1/2$, there is only one possible J, which is $J = 1/2$. For $L = 1$, there are two, $J = 1/2$ and $J = 3/2$. The energy splitting between $L = 0, J = 1/2 \leftrightarrow L = 1, J = 1/2$ is the D_1 line, whereas $L = 0, J = 1/2 \leftrightarrow L = 1, J = 3/2$ is the D_2 line. An example of the hyperfine structure for the D_2 line for cesium is in Fig. 2.

Let's consider an atom in an electromagnetic field, that is of a much longer wavelength, than is the size of the atom. In this case, the electromagnetic field is roughly the same throughout the whole atom. Therefore, this electric potential can be approximated as only time dependent. In the first order time-dependent perturbation theory, we can derive these electric dipole transitions, marked as E1. They are the most probable ones in respect to other light induced atomic transitions like magnetic dipole M1, electric quadrupole E2 and higher multipole transitions (usually by at least a factor of 100; we get these by considering also the higher orders in expansion in position – by saying that the atom is not small in comparison to light wavelength [12]). These higher orders are therefore, due to their improbability in relation to the electric dipole transitions, called forbidden transitions. To see what transitions are allowed we can look at selection rules.

What we now want to do is get the electron into one of the excited states, that have a forbidden transition back into the ground state. Therefore, it cannot decay via allowed transitions. Such states are called metastable. Just "meta" because they eventually do decay. Either via collisions, multiphoton transitions or forbidden transitions. Stil, such decays are, as said before, much slower than the allowed ones. We can see how this can function as a storage state in our quantum memory.

2.3 Electromagnetically Induced Transparency

There are multiple schemes for the realization of quantum memory on atomic vapors. One of them, that is widely used and is useful for understanding the type of memory we are interested in, is Electromagnetically Induced Transparency (EIT). In it, we commonly use a lambda scheme of hyperfine states. It is called like that, since the states and their connecting (allowed) transition resemble an upper-case Greek letter lambda (Λ) as in Fig. 3.

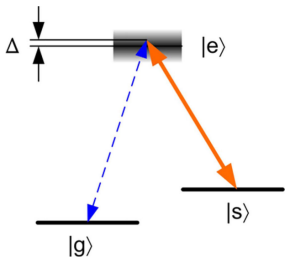

Figure 3: The Λ EIT scheme with detuning Δ. Signal is blue and control is orange. [13]

Let's call these states ground $|g\rangle$, excited $|e\rangle$ and storage $|s\rangle$. Their energies are $E_g = 0 < E_s < E_e$ and the two allowed transitions are $|g\rangle \leftrightarrow |e\rangle$ and $|e\rangle \leftrightarrow |s\rangle$. We can pump atoms between these states with two laser beams, which we can call "signal" and "control". Signal handles the $|g\rangle \leftrightarrow |e\rangle$ transition, has frequency ω_s and a Rabi frequency Ω_s, whereas control handles the other, $|e\rangle \leftrightarrow |s\rangle$ transition, has

frequency ω_c and a Rabi frequency Ω_c. Generally, the frequencies of these beams are offset by some detuning Δ from the actual frequency that gets them to state $|e\rangle$, that is the resonant frequency. After all, there is some non-zero linewidth for a transition and later we will see, that by changing this detuning Δ, we can have different processes present. We can control detuning to a very high precision.

The Rabi frequency is $\Omega = \vec{p} \cdot \vec{E}_0/\hbar$, where $\vec{p} = \langle a|e\vec{r}|b\rangle$ is the electric dipole moment ($e\vec{r}$) matrix element between two states $|a\rangle$ and $|b\rangle$, $\vec{E}_0$ the constant amplitude of the light's sinusoidal electric field strength and $\hbar$ the reduced Planck constant. Basically, the Rabi frequency gives us the coupling strength between the light field and the state transition. The higher this frequency is, more possible is the transition. There are all in all three possible 3-state schemes of two transitions, the other two being vee (V) and ladder (Ξ), again, because of the way they look – see Fig. 4.

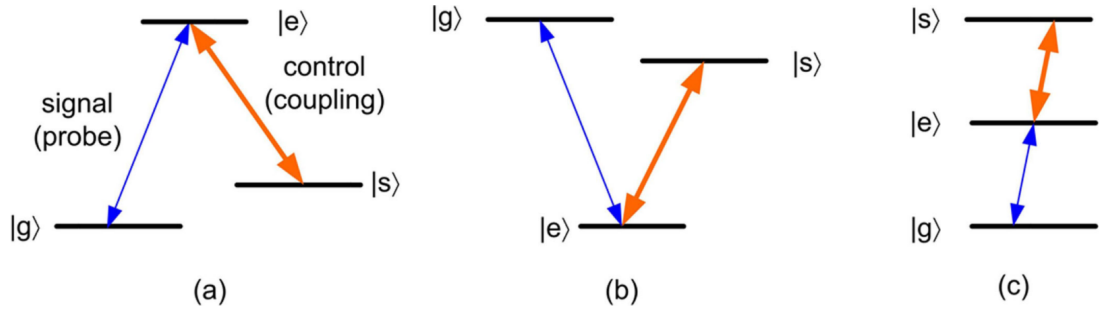

Figure 4: Three-level schemes: (a) Λ, (b) V and (c) Ξ [13].

Let's look at what is happening with this Λ system, if we shine either of the light beams upon it. At the beginning, if the system is in thermal equilibrium, it is most probably in its lowest energy state $|g\rangle$. How probable the other states in thermal equilibrium are, is given to us by the Boltzmann distribution $p_i \propto \exp(-\beta E_i)$, where p_i is the probability of the system being in state $|i\rangle$ with energy E_i at the temperature T of the ensemble (packed in the standard notation $\beta = (k_B T)^{-1}$ with k_B being the Boltzmann constant). More useful is perhaps looking at these ratios for different states, where we can say that the states' $|i\rangle$ and $|j\rangle$ populations N_i and N_j are in a ratio $N_i/N_j = \exp[-\beta(E_i - E_j)]$. In hot vapors it is then quite possible, if some state is close to the ground state, that some atoms are in that other state. At room temperature, the thermal energy is $k_B T = 4.14 \cdot 10^{-21}$ J. The cesium $6^2S_{1/2}$ hyperfine energy splitting (depicted in Fig. 2) is $6.09 \cdot 10^{-24}$ J [14]. In this example, the two hyperfine states' population ratio is at practically the high temperature limit, with a value $N_{F=3}/N_{F=4} = 1.0015$.

If we introduce now the resonant signal light field into the system, there is some absorption of the ground state into the excited state. This means that if the ensemble has a high enough optical depth, this light is quite soon completely absorbed until we get to a new steady state. There is now absorption and stimulated emission in transition $|g\rangle \leftrightarrow |e\rangle$ and some spontaneous emission from state $|e\rangle$ to both $|g\rangle$ and $|s\rangle$.

If we would initially only introduce the control field into this system, in the ideal case of no atom population in state $|s\rangle$, nothing would happen. The control field would just go through the ensemble. But as shown with the above quick calculation, there is always some population in all the states of a hyperfine splitting. We can circumvent this by prior repumping of atoms from one state to another.

But if we turn on both light fields, signal and control, these effectively make up a new field. These two fields interfere destructively, so we do not get any of the above transitions, but we now have a chance for a two-photon transition, if their frequency difference matches with this two-photon resonance $|g\rangle \leftrightarrow |s\rangle$ at ω_{gs}, which is forbidden for a one-photon transition. This means that for the latter transition, for beams signal and control, it is now for a part of the fields that are at the right frequencies $\omega_s - \Delta - (\omega_c - \Delta) = \omega_s - \omega_c = \omega_{gs}$ (with low enough linewidth this can be the whole light field) as if they did not exist. So in the middle of what was an absorption peak in the case of just the signal beam is now a dip that goes to zero (there, absorption is zero). The Kramers-Kronig relations for the complex refractive index $\mathcal{N}(\omega) = n'(\omega) + in''(\omega)$, with the imaginary part $n''(\omega)$ being absorption and the real $n'(\omega)$ refractive index, now tell us the refractive index must there be very steep, as visible in Fig. 5. The group velocity [15]

$$v_g = \frac{c_0}{n' + \omega \frac{dn'}{d\omega}}, \tag{1}$$

with c_0 being the speed of light in vacuum, thus decreases (for derivative $dn'/d\omega$ increases). Consequently this spatially compacts the incoming light in the ensemble by the ratio c_0/v_g, ideally with its whole pulse length within the ensemble. We can associate this slow propagation of light with a quasiparticle, called the dark-state polariton.

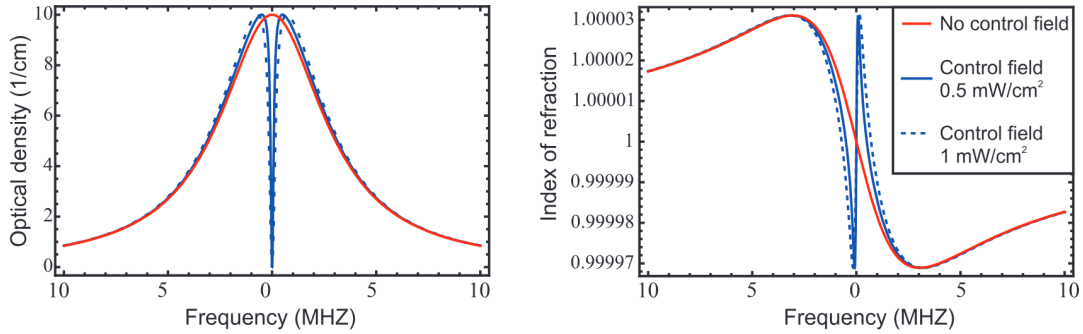

Figure 5: The complex refractive index in EIT medium: left imaginary, right real part [16].

From the above desription of the $|g\rangle \leftrightarrow |s\rangle$ transition we can say that its coherence decay is slow. The popagating signal (as light) can be represented with the electric field operator $\hat{E}(z,t) = \sum_k a_k(t)e^{ikz} + c.c.$, summed over the free space modes of wave vectors k, position z and bosonic operators a_k (c.c. is the complex conjugate of the first term). In a small ensemble volume of N_z atoms, we can describe the medium with slowly-varying collective atomic oprators, called "spin-waves". At the higher, signal frequency ω_{ge}, with an operator $e(\hat{z},t) = N_z^{-1}\sum_{j=1}^{N_z}|g_j\rangle\langle e_j|\,e^{-i\omega_{ge}t}$ and at the lower frequency ω_{gs} with an operator $\hat{s}(z,t) = N_z^{-1}\sum_{j=1}^{N_z}|g_j\rangle\langle s_j|\,e^{-i\omega_{gs}t}$. The control field is here treated classically, because it is much stronger than the signal field. Signal is assumed to be weak for the sake of simplicity in derivation. The time evolution of this system [17] gives us a superposition $\hat{\psi}$ of the photonic and the atomic spin-wave parts as

$$\hat{\psi}(z,t) = \cos(\theta)\hat{E}(z,t) - \sin(\theta)\sqrt{N}\hat{s}(z,t); \quad \cos(\theta) = \frac{\Omega_c}{\sqrt{\Omega_c^2 + g^2N}}, \sin(\theta) = \frac{g\sqrt{N}}{\sqrt{\Omega_c^2 + g^2N}}. \tag{2}$$

Interestingly enough, the dependence on $\hat{e}$ has fallen out. This also means, that in this state $\hat{\psi}(z,t)$, there cannot be any spontaneous emission, which is quite welcome. We now see, that when we turn the control beam (Ω_c) off, the beam is completely inscribed onto the atoms as a spin-wave: $\hat{\psi}(z,t)|_{\Omega_c=0} = -\sqrt{N}\hat{s}(z,t)$, whereas in a strong control beam, not many spin-waves are created: $\hat{\psi}(z,t)|_{\Omega_c >> g\sqrt{N}} \sim \hat{E}(z,t)$. These cases are visible in Fig. 6.

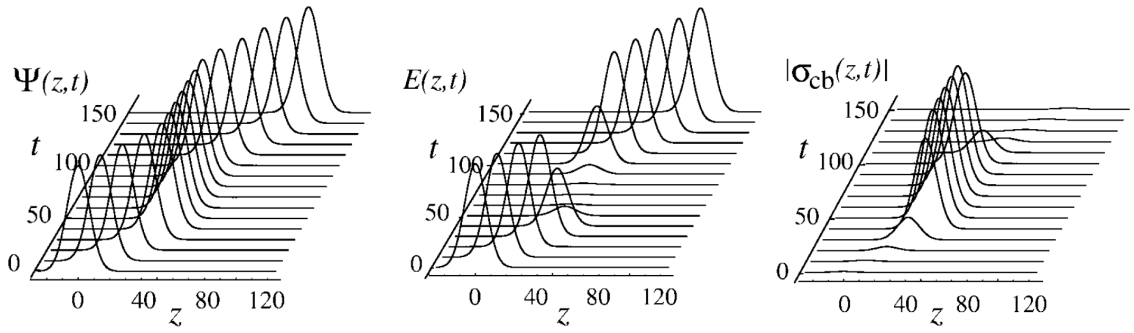

Figure 6: The superposition $\hat{\psi}$ (left), its photonic part (middle) and its spin-wave part (right) [18]. From the slope of the graph we can see the slow speed of the photonic part and the stationary spin-wave part of the superposition.

Like this we have just written the polarization of the signal beam onto the atoms as a spin-wave. It is

quite evident from (2) that when we want to transform the spin-wave back into its photonic state, we must only turn the control beam back on. In practice, vertical and horizontal light polarization, corresponding to different qubits, can be 'saved' like this. Now that I have introduced the more technical explanation of 'saving' light pulses on atoms as spin-waves, with a representative realization, we can move forward to Raman type DLCZ quantum memory.

2.4 Raman type memory

First off, what is a Raman transition? We might quickly think of the most widely represented Raman transitions in molecules, where some frequency of light is absorbed by the molecule, which then "uses" this energy for transfering into higher vibrational modes, and because of this extra intremediate step, reemits the light in some other, lower frequency. In atoms we clearly do not have such vibrational modes. But Raman scattering is in atoms, like in molecules, an inelastic process. From here on out I will of course be refering to the scattering in atoms.

Looking again at a 3-level system and the possible transitions between them in dependence of the frequency of the incident light ω, we can generally say, elastic or inelastic scattering can occur. There are four possible scattering types, as seen in Fig. 7, from left to right:

- Let the system at first be in its ground state $\omega_1 = 0$. If light, that is off-resonance for any possible transition, scatters on this system, and the scattered light is of the same frequency, this is Rayleigh scattering, which is elastic.
- If the incident light is within some linewidth of a possible transition ω_l, which then happens, this is fluorescent scattering (in this case it would appear as if some energy was missing, but this is within the energy uncertainty of the transition) [19].
- Moving further away from this transition ω_l and if this light has its frequency higher than another possible transition ω_f, more possible than the fluorescent one becomes the Raman transition. Here then, if incoming light had frequency ω, the scattered light will be of frequency $\omega - \omega_f$, usually called Stokes light (in the case of $\omega - \omega_f < 0$ it is called anti-Stokes light).
- If ω is close to ω_l, this Raman transition can still happen, but it is then called fluorescent Raman and is less likely than the fluorescent one.

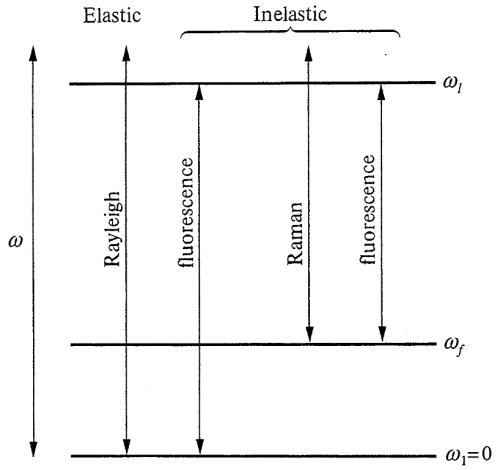

Figure 7: Types of scattering: elastic Rayleigh, inelastic fluorescent, Raman and fluorescent Raman. Incident frequency here is ω on the very left [19].

We can now consider a Raman transition on the aforementioned Λ system from the chapter on EIT. If we shine on it with an off-resonant signal beam (off-resonant in relation to $|g\rangle \leftrightarrow |e\rangle$), there is a high probability it will not scatter, but a small one that there will be a (off-resonant) Raman transition [20]. There is no other outside light field in this system (only signal, no control).

In an ensemble of such atoms, we are very interested in the Stokes photons that propagate in the same direction as the signal beam. They are, being co-propagating, uniquely correlated with a mode of symmetric colective spin-wave $\hat{S} = \sqrt{N_{\text{ens}}}^{-1} \sum_{j=1}^{N_{\text{ens}}} |g\rangle_j\langle s|$, where N_{ens} is the number of atoms in the ensemble. See here, there is only one $\langle s|$ in question. When a single Stokes photon is emitted in the direction of the signal beam (so the result of our operator $\hat{S}$ acting on the collective ground state) from within the ensemble, the resulting state is $\hat{S}^\dagger|0_{\text{ens}}\rangle$, since its previous state was the ensemble ground state $|0_{\text{ens}}\rangle = \otimes_j |g\rangle_j$, with this last term being the tensor product over all the atoms' ground states (standard notation in Fock space).

Moving on, we should also define a single mode (at Stokes frequency) bosonic operator $\hat{a}_{\text{pho}}$ for the Stokes photons and a corresponding vacuum state for this mode $|0_{\text{pho}}\rangle$. Then, because this Raman transition does not happen every time, the forward-scattering Stokes mode, after passing the signal field through the atoms, can be written as a superposition $|\Phi\rangle$ of "nothing happened", "one Stokes photon was emitted" and "more Stokes photons were emitted" (these terms presented as $O(p_c)$, since they happen with higher powers of probability than the single-Stokes-excitation probability p_c)

$$|\Phi\rangle = |0_{\text{ens}}\rangle|0_{\text{pho}}\rangle + \sqrt{p_c}\, \hat{S}^\dagger \hat{a}^\dagger_{\text{pho}}|0_{\text{ens}}\rangle|0_{\text{pho}}\rangle + O(p_c). \tag{3}$$

Afterwards, we can shine upon it the control beam $|s\rangle \leftrightarrow |e\rangle$. Via an opposite Raman process with the emmision of the anti-Stokes photon ($|g\rangle \leftrightarrow |e\rangle$) the ensemble returns to the collective ground state. Here, we could ask ourselves what if the anti-Stokes photon is now absorbed by the ensemble? With having enough off-resonant control beam, such a transition is quite unlikely, but closer to it we return to the EIT regime. The presence of the control beam would in this case again cause transparency for the signal photon, which is otherwise in current context called the anti-Stokes beam.

The take-away of this section is that a spontaneous Raman transition is ideal for generating entangled pairs of Stokes photons and excited collective atomic states (or in a reverse process anti-Stokes photons and ground collective atomic states). This is because in this process the excitation (or relaxation) of collective atomic state must directly correspond to a spontaneous emission of a Stokes (or anti-Stokes) photon. The collective atomic states are coherences between the ground and storage atomic states, so spin-waves, more pictorially presented in the previous section. Due to entanglement of photonic states with, essentially their corresponding quantum memories, we will show in the next chapters that we can make collective operations on such photons emitted from separate quantum memories, which in turn directly affects the correlations between the two corresponding separate quantum memories. We will show that we can actually entangle separate quantum memories in precisely this way.

3 Entanglement of Atomic Ensembles

3.1 Entanglement by a Beam Splitter

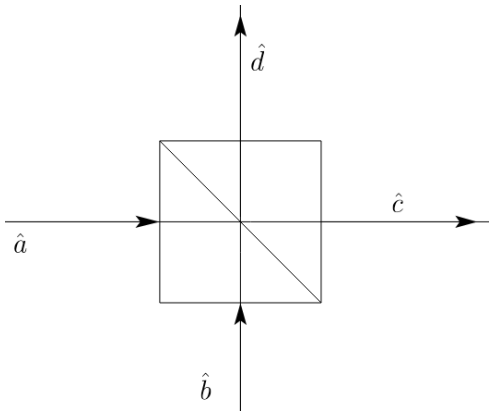

Figure 8: Entanglement by a beam splitter [21].

We will be working with photonic states and so we will need a way for entangling photons. This can be done for example with a 50-50 beam splitter (BS) (half probability for transmitting, half for reflecting),

depicted in Fig. 8. If we align two light beams onto this BS, looking at its exit, we cannot tell, which came from which beam. Sending in classical light is not enough, though. For successful entanglement it needs to be sufficiently non-classical [21]. If we align the non-classical sources correctly, so that we can know for neither, by measuring flight-time, where it came from (plus anulling all other possible separating qualities), it is impossible on the other side of the BS to know what path the photon came from. So the two possible paths are entangled. By extension also their sources are entangled. In fact, if only one photon is measured at the exit of the BS, we know that it came from exactly one source, but we do not know which one, and so we entangle two states of the source. This is what we will use for entangling our sources, the atomic ensembles. Again, in this way we cannot entangle photons, since their measurement at the other side of the BS destroys their states. We entangle their paths, so by extension their sources, which was exactly our goal.

3.2 First Entanglement

Now, let us put all the previously discussed topics together. First, let's entangle a pair of quantum memories, as shown in Fig. 9. Take two atomic ensembles of atoms with the Λ transition scheme, such as considered throughout this seminar. Behind each of them we put a filter, that gets rid of the signal light, but lets the Stokes light pass. After this filter we bring both beams onto a BS. One on one side, other on the neighbouring one. Both can either be reflected or transmitted. But anyways, they can come out the BS only in two paths. On each of these two we position a single photon detector, D_1 on first, D_2 on second.

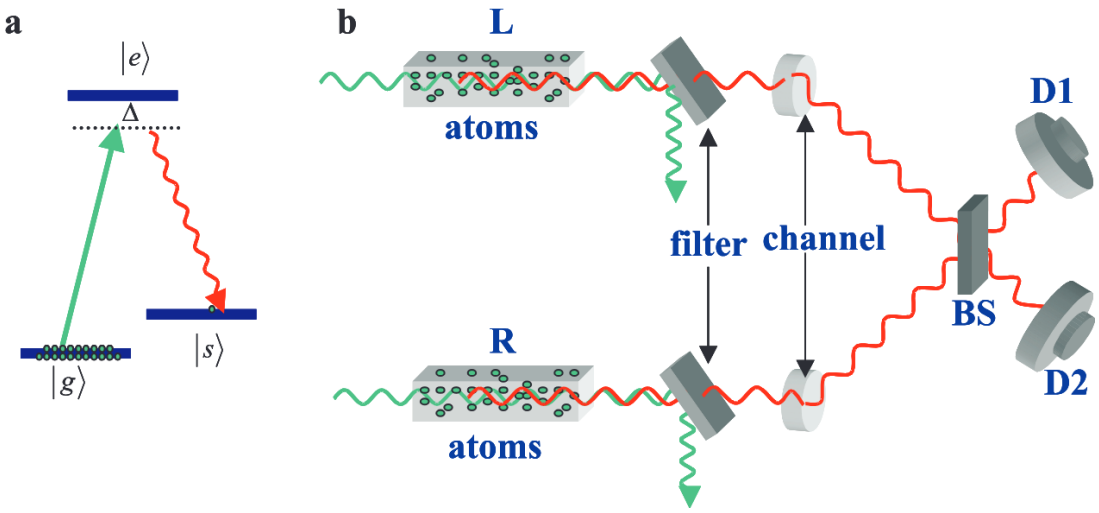

Figure 9: Creating entaglement between two memories. (a) the Raman transition – signal green and Stokes squiggly red, (b) a scheme of the experiment [22].

At the same time, we send in each of the ensembles the Raman signal pulse. It must be short enough, that we excite on average (average over number of experiments) much less than one Stokes photon, this probability being $\sqrt{p_c}$ from (3).

Intermezzo: It is easier to understand this plan of ours, if we say that the Stokes photon is either emitted and then definitely detected on one of the detectors, or is not emitted and then of course is not detected. In reality the Stokes photon can be scattered off of our symmetric mode, we can detect dark counts on the detectors or we can otherwise experience other real world problems. But as said, we think of the ideal case.

Then, the probability of detecting the Stokes photon on D_1 *or* on D_2 is p_c. Considering each such experiment is essentially a Bernoulli trial [23, 24] with probability p_c for success, these experiments constitute a binomial distribution. And the binomial distribution says that if we make this test N times, it is most likely we will succeed $p_c N$ times. Therefore, we need to roughly repeat this experiment $1/p_c$ times to get a successful one. The caveat is that if p_c is too high, there is a higher possibility of detecting more than one photon. If we do not detect any, we know neither ensemble is excited. If we detect two, (probably; with higher p_c maybe one is doubly excited – still bad) both ensembles are excited. In both

these cases we know (probably) everything about them. But if we detect only one, they are entangled as per the reasoning from the above subsection on BS entanglement.

Denoting the memories as left $|\Phi\rangle_L$ and right $|\Phi\rangle_L$, given by (3), the whole system is described by the state $|\Phi\rangle_L \otimes |\Phi\rangle_R$. A click on either detector D_1 *or* D_2 is a measurement of the combined excitations from the two memories, represented by $\hat{a}^\dagger_{pho,+}\hat{a}_{pho,+}$ or $\hat{a}^\dagger_{pho,-}\hat{a}_{pho,-}$, where $\hat{a}_{pho,\pm} = (\hat{a}_{pho,L} \pm e^{i\zeta}\hat{a}_{pho,R})/\sqrt{2}$ and ζ is some (normally real) unknown phase difference between the two channels. This is quite obvious, since detecting a photon destroys its photonic state. Detecting a photon means we have a state, described by the photonic annihilator $\hat{a}_{pho,+}$ or $\hat{a}_{pho,-}$ operating on the state $|\Phi\rangle_L \otimes |\Phi\rangle_R$. Glancing at equation (3) this operation annihilates the ground state and lowers the excited state. This makes sense – a ground state could not have emitted the photon. We are left with

$$|\Psi_\zeta\rangle^{\pm}_{LR} = \frac{1}{\sqrt{2}}(\hat{S}^\dagger_L \pm e^{i\zeta}\hat{S}^\dagger_R)|0_{\text{ens}}\rangle_L|0_{\text{ens}}\rangle_R. \tag{4}$$

The $\pm$ states can be transformed into each other by a local phase shift. Finally, with the presence of noise, specifically the "vacuum" coefficient c_0, determined by the detector dark counts, we get the state of the pair for $|\Psi_\zeta\rangle^{+}_{LR}$ (similar for $-$), given as a density matrix (pure states cannot account for the losses), as

$$\rho_{LR}(c_0,\zeta) = \frac{1}{c_0+1}(c_0\,|0_{\text{ens}}0_{\text{ens}}\rangle\langle 0_{\text{ens}}0_{\text{ens}}|_{LR} + |\Psi_\zeta\rangle\langle\Psi_\zeta|^{+}_{LR}). \tag{5}$$

This state is called an effective maximally entangled (EME) state.

3.3 Entanglement Purification

The fidelity F of the EME state depends on uncertainties of our detection. The sum efficiency of photon loss due to coupling of light, channel attenuation, spontaneous emission, detectors etc. is η_p. If dark counts per entanglement preperation time have a probability p_{dc} and since the expected number of required preperations for one count is (a couple paragraphs ago), now accounting for photon loss, $1/\eta_p p_c$, the vacuum coefficient is $c_0 = p_{dc}/\eta_p p_c$. But c_0 is normally much smaller than one, because typically, $p_{dc} << p_c$. A more serious concern is having two excitations, whereas only one is detected. Probability of this is given by p_c, which gives us the fidelity imperfection $\Delta F \sim p_c$. If the time for entanglement preparation (one experiment, successful or not) takes time Δt, then the time needed to get to a successful preparation is $T_0 \sim \Delta t/\eta_p p_c$. We see that we can decrease the excitation probability p_c and with this improve the fidelity F of the EME state, but for the price of longer total preparation time T_0. This is how we can purify our state.

3.4 Entanglement Swapping

We now want to entangle the already connected pairs of entangled memories into a long chain. We call this entanglement swapping and it can be seen in Fig. 10.

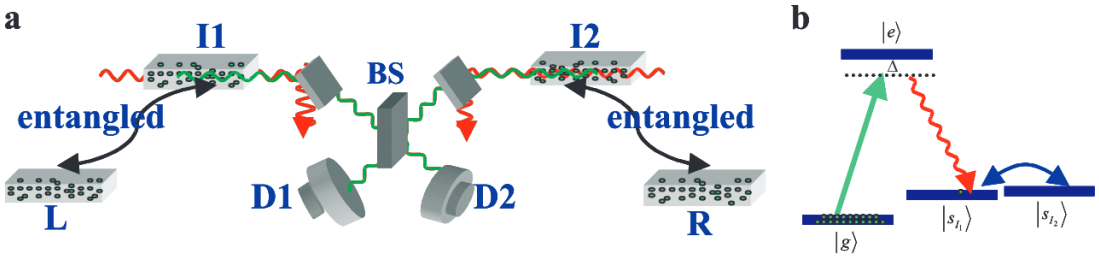

Figure 10: Entanglement swapping between two entangled memory pairs. (a) the scheme of the experiment – R from one previous pair is I_1, L from another previous pair is I_2, (b) the Raman transition – anti-Stokes green and control squiggly red, this one visibly pumped in on (a); $|s_{I_1}\rangle$ and $|s_{I_2}\rangle$ represent previous entanglement [22].

What we do, is take one entangled memory from each of two neighbouring entangled pairs. We now shine a control field (where before Stokes field was created) into both of them at the same time. With

some possibility on each of the memories, the Raman transition happens and the anti-Stokes photon can be emitted. Moving on is essentially the same as with the creation of the first entanglement pairs. Here is a chance, neither of the two ensembles will emit a photon, since in the previous entanglement just one photon was shared between a pair. Or we can get now two photons, equally likely. But this would again mean, that we now know everything we can about the pair of these second entanglement pairs. Actually, we also now know what the states of the other two memories from the first entanglement are. What we need is exactly one detection, as before. If we get anything else, we must repeat the first entanglement and the second one for these four memories. We should also be making these second entanglements in segments, since if they fail, the entanglement of the possible other connected second entanglements fails as well. This is where the polynomial whole channel entanglement preparation comes from. When we do succeed, however – the first and the last memories in the whole chain are entangled too. As we wanted.

4 Conclusions

The aim of this seminar is to explain the workings of the DLCZ protocol. To explain it on a Masters' studies level, some extra steps are required. We see how quantum memories are constructed, how one of its most well known representatives, the EIT, behaves and upon this one we expanded into the operation of the DLCZ memory, utilizing the spontaneous Raman transition. Perhaps calling it a DLCZ 'memory' in this context is a bit misleading, because here, its atomic 'quantum memory' part is equally important to its simultaneously emitted photonic part. Specifically we are exploiting the entanglement of these two parts. For this purpose I have shown, how central to its usefullness is the fact, that at a transition it gives off a photon, announcing success at writing or reading the memory. Moving on, this can then be used for pair-wise entanglement of multiple memories on the basis of a beam splitter entanglement. In the end, we have constructed long distance entanglement in a time that scales only polynomially with the distance. This protocol could prove pivotal in the developement of the quantum internet and for other uses of long distance quantum state transfer as for quantum key distribution or quantum state teleporation.

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