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Seminar 2

Spatial light modulation and generation of box traps

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ABSTRACT

Ultracold atoms are used to study some of the most fundamental concepts in quantum mechanics. To do this, scientists create specific conditions for the atoms to exist in, which typically involves complex optical setups. Until recently, it was extremely difficult to create an optical box potential, that is, an optical trap that has areas with a constant potential surrounded by steep walls. However, with the advancements in light manipulation using microelectromechanical devices and liquid crystals, it is now possible to have better control over the light used to create the potential field. This new level of control allows more flexibility and precision when working with ultracold atoms.

Contents

1 Introduction

In experiments with ultracold atoms, one can study the fundamental behaviour of different quantum systems. The evolution of a quantum system is determined by the potential in which it resides. To create such a system, a cloud of ultracold atoms is prepared and placed in different types of electromagnetic and optical traps that create a potential, hold the cloud in place, and cool the atoms down. One of the simplest potentials that can be created is a box trap. A box trap is a potential with a flat bottom surrounded by steep walls. Even though it is a simple potential, it allows for a wide range of experiments to be conducted. By studying the behaviour of ultracold atoms in a box trap, we can gain insights into a variety of quantum phenomena, such as quantum tunnelling and the behaviour of interacting many-body systems [1]. The simplicity of the box trap potential makes it a valuable tool for exploring fundamental quantum physics.

2 Creation of box potentials

In order to understand how a box trap potential is created, we first need to understand how optical trapping works. To this end, we'll explore a phenomenon called optical tweezing, which occurs when the laser beam creates a gradient in the refractive index of the medium around the particle. Furthermore, we will need to understand how light propagates in space. Hence we will describe the simplest mode of laser beam propagation. We will take a look at the potential made by a laser beam, followed by a description of how special lenses can be used to make a box potential. Next, we will introduce the concept of Fourier optics, which will serve as the backbone of a useful optical technique called spatial light modulation.

2.1 Optical trapping

The potential created by light is directly related to the force that light exerts on a particle. The force is caused by the change in momentum of a photon being scattered by the particle [2]. We can start by describing an atom from which we scatter a single photon with wave vector $\mathbf{k}$. After the scattering process, we get a photon with a new wave vector $\mathbf{k}'$. We will assume that no energy was transferred, e.g. the atom was not excited to a higher state. Consequently the following stands $|\mathbf{k}| = |\mathbf{k}'|$. In simpler terms, this means that the only thing that changed after the scattering is the trajectory of the photon. We can describe the momentum of the atom using its wave vector as $\mathbf{p}_{atom} = \hbar \mathbf{k}_{atom}$. Using conservation of momentum we can write $\mathbf{p}_{atom} = \hbar(\mathbf{k} - \mathbf{k}')$. Here we have assumed that the atom had zero momentum before the scattering process. Force can be expressed as a time derivative of momentum. For now, let us assume that the scattered photon will always travel along the same path. The force exerted on the atom can be written as:

$$|\mathbf{F}| = 2\frac{P}{c}\sin\frac{\theta}{2}. \tag{1}$$

Here we have assumed that θ is the angle between the incident and scattered trajectory and P is the optical power carried by the photons. Until now, we relied on the assumption that trajectory of the incident photon as well as the scattering angle were identical for every photon. This assumption is false since scattering directions are randomly distributed. The distribution is dependent on the interaction between the photon and the scattering target. It is still possible to find an average scattering direction, which depends on the trajectory of the initial photon, its wavelength and the properties of the scattering target. The force can then be expressed as a sum of all force contributions from all possible directions of the incident photons.

Next, let us treat the atom as s a small spherical lens (see Fig. 1). We can now separate two different scenarios just based on symmetry considerations.

Let us first consider the case where the atom lies in the center of the beam. In this case, we can assume cylindrical symmetry. Consequently all contributions to force that are perpendicular to the laser beam will cancel out. Next, let us consider the case where the atom is not in the center of the beam, and we do not have cylindrical symmetry. In this case, the force perpendicular to the beam may not cancel out. The direction of the force depends depends on the gradient of light intensity as well as the interaction between the light and the particle. If the particle causes light to converge after the scattering, the force will point towards higher intensities, e.g. the particle will move to the center of the beam. In case where the scattering causes the light to never converge, the force will point towards lower intensities, e.g. the particle will move away from the beam. In case of the ultracold atoms this simply means that we can make either an attractive or a reflective potential. The interaction depends on the wavelength of the light in relation to the energy levels of the selected atom.

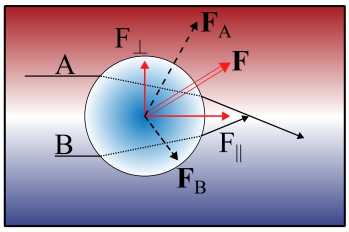


Figure 1: Beams A and B exert force onto a particle. If the light intensity field varies across the particle, beams A and B make different contributions to the force resulting in a non-zero force perpendicular to the beam. Force parallel to the beam is always present.

Regardless of the symmetry and interaction between the particle and light, force parallel to the beam will always point in the direction of the beam. This can be explained by looking at two limiting scenarios, first where all of the light reflects off the atom as if it were a flat mirror and the second where all of the light is transmitted along the trajectory of the incident light.

2.2 Light propagation

Since light is an electromagnetic wave, it can be described using the following equations, which stem from the Maxwell's equations:

$$\begin{aligned}\frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2} - \nabla^2\mathbf{E} = 0,\\ \frac{1}{c^2}\frac{\partial^2\mathbf{B}}{\partial t^2} - \nabla^2\mathbf{B} = 0.\end{aligned} \tag{2}$$

Here c represents the speed of light in the medium and can be expressed as $c = (\mu_0\varepsilon_0\varepsilon\mu)^{-1/2} = c_0(\varepsilon\mu)^{-1/2}$. Here ε is the relative permittivity and μ is the relative permeability and c_0 represents the speed of light in vacuum. We are mostly interested in results describing the behaviour of light either in a vacuum or in air. Hence we can assume that $c \approx c_0$. The simplest solution to Eq. (2) is simply a plane wave:

$$\mathbf{E}(\mathbf{r},t) = \mathbf{E}_0 e^{i\mathbf{kr}-i\omega t}. \tag{3}$$

Where $\mathbf{k}$ is the wave vector of the field and $\omega = |\mathbf{k}|c$ is the angular frequency of the field. Another important solution of Eq, 2 is the spherical wave:

$$\mathbf{E} \propto \frac{1}{|\mathbf{r}-\mathbf{r}_0|}e^{ik|\mathbf{r}-\mathbf{r}_0|}. \tag{4}$$

Here $\mathbf{r}_0$ is the location of the centre of the spherical wave and k is a scalar wave vector. Since Eq. (2) are linear, any sum of both the plane waves and the spherical waves will also be a valid solution.

2.2.1 Gaussian beams

To understand why laser light is insufficient when creating a box potential, it is important to be able to describe a typical laser beam. Consequently, we will look into a Gaussian beam. It is important to note that lasers often have different beam shapes, which can typically be converted into a Gaussian beam using different methods, e.g. coupling into a single-mode optical fiber. Furthermore, a Gaussian beam is the lowest order beam profile. We can start by splitting the electric field into a rapidly and a slowly varying part as : [3]:

$$\mathbf{E} = \mathbf{E}_0\Psi(\mathbf{r},z)e^{ikz-i\omega t}. \tag{5}$$

Here we assumed that the field propagates along the z axis. We defined $\mathbf{r}$ as a vector in the xy plane, and Ψ as a slowly varying modulation of the field. By inserting $\mathbf{E}$ into Eq. (2) along with approximation $\partial^2\Psi/\partial z^2 \approx 0$, we get the paraxial equation:

$$(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2})\Psi + 2ik\frac{\partial}{\partial z}\Psi. \tag{6}$$

One of the solutions of the paraxial equations is a Gaussian beam

$$\mathbf{E}(\mathbf{r},z,t) = \mathbf{E}_0 \frac{w_0}{w} e^{ikz-i\omega t} e^{-r^2/w^2} e^{ikr^2/2R} e^{i\eta(z)}, \tag{7}$$

which describes a typical laser beam. Here we defined $w^2 = w_0^2[1+(z/z_0)^2]$, $R = z[1+(z_0/z)^2]$ and $\eta(z) = \arctan(z/z_0)$. Here z_0 and w_0 are parameters of the shape of the beam and are related through $z_0 = \pi w_0^2 \lambda^{-1}$. We are mostly interested in the intensity profile of the beam, which can be written as:

$$I = |\mathbf{E}(\mathbf{r},z,t)|^2 = I_0 \frac{w_0^2}{w^2} e^{-2\frac{r^2}{w^2}}. \tag{8}$$

A plot of intensity can be seen on Figure 2. We can see that the intensity is a Gaussian function of r with width $w(z)$. Parameter w_0 simply tells us the width, when the beam is the most narrow at $z = 0$.

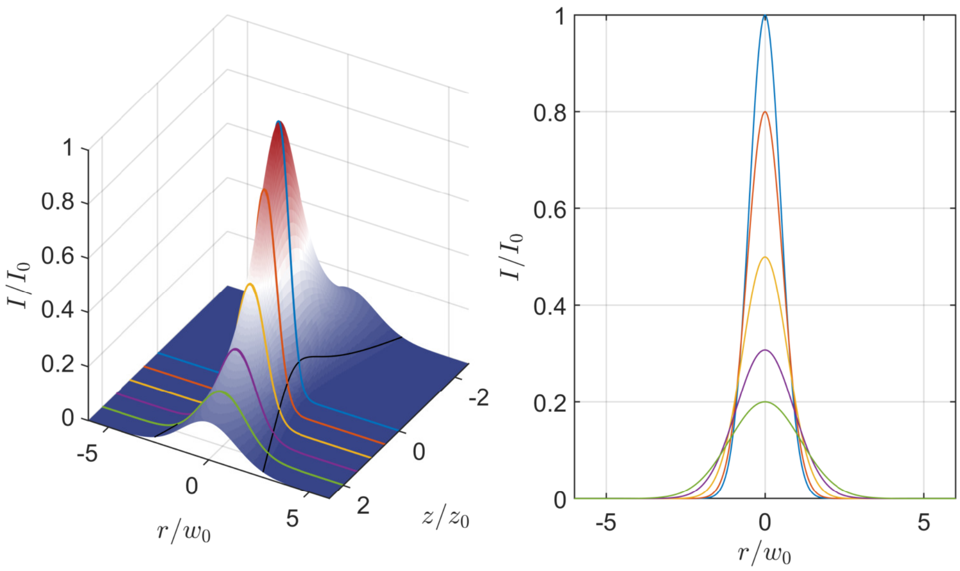


Figure 2: Intensity profile of a Gaussian beam. Black line depicts width of the beam w as a function of z. Coloured lines show intensity profiles at different depths. These are shown on the plot on the right with matching colors.

2.2.2 The trapping potential of Gaussian beam

In chapter 2.1 we learned that the potential created by light is proportional to the intensity of the light. In this section we will first take a look at why the potential generated by a typical laser beam is harmonic. As stated previously a typical laser beam is simply a Gaussian beam. Using Eq. (8) we can write the following approximation using a series expansion of an exponential function:

$$U_d \propto I = I_0 \frac{w_0^2}{w^2} e^{-2\frac{r^2}{w^2}} = I_0 \sum_{n=0}^{\infty} \left(2\frac{r^2}{w^2}\right)^n \frac{1}{n!} \approx I_0 \left[1 + 2\frac{r^2}{w^2}\right]. \tag{9}$$

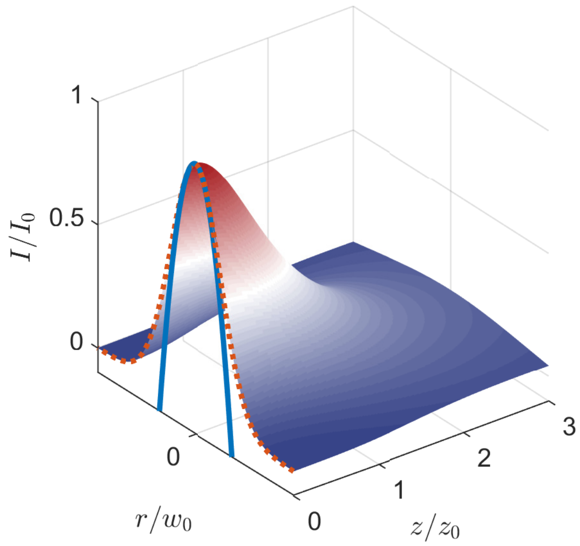
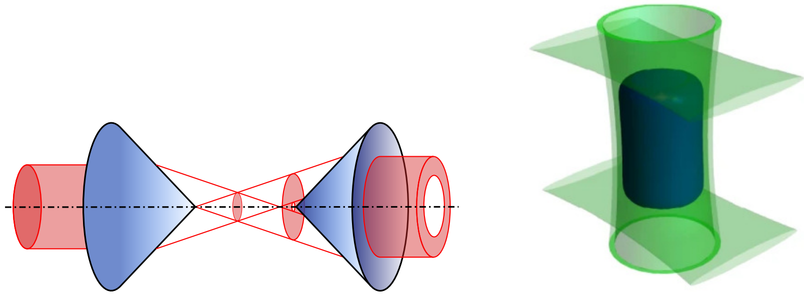


Figure 3: Comparison of intensity profile of a Gaussian beam with harmonic approximation (light blue line) given in Eq. (9).

As can be seen in Figure 3, we can approximate the optical potential as some constant potential together with a harmonic potential proportional to the square of the distance from the centre of the beam. As stated in the beginning, this produces an uneven density of the particles, since they all tend to move toward the centre of the beam. Hence it is impossible to make a box potential using a simple Gaussian beam. It is important to note that other sources such as gravity also contribute to the force on the particle.

2.3 Creating box potentials using axicons

An axicon is a type of lens which has a flat surface on one side and a conical surface on the other side. It can be described purely in terms of geometric optics. As can be seen in Figure 4a an axicon produces an area where an intensity profile is shaped like a ring, which can serve as an optical trap if it is combined with auxiliary beams as can be seen in Figure 4b. When using a single axicon, the size of the ring will depend on the distance from the axicon. By using a pair of axicons the size of the ring can be fixed to a desired dimension. An important aspect to consider is the fact that when using axicons only cylindrical box traps can be made. Often rectangular traps are preferred, which require more sophisticated solutions. Here, we have to mention that when using axicons, the light is chosen such that the interaction is repulsive. Henceforth the annular ring from the axicon and two auxiliary beams are used to make the walls of the trap.



(a) (b)

Figure 4: (a) Schematic diagram of two axicons. By shining a beam of light through an axicon an annular ring is created, which expands with increasing distance. If two axicons are placed opposing each other, a fixed sized ring can be created. This can be used to create optical traps. (b) A diagram of a three dimensional optical trap using a light with a ring shaped intensity profile and 2 auxiliary beams. Image taken from [1]

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2.4 Spatial light modulation

Spatial light modulation (SLM) is a relatively simple technique, where we modulate the intensity and the phase of an incident beam. The modulation can vary in space, and it is typically done on a plane. When writing equations we will use superscript S to denote that we are referring to a property in the SLM plane and superscript O in the image/optical plane. To model the behaviour of the output light we will use the Huygens-Fresnel principle. This will allow us to describe a modulated electric field E^O, which is a result of electric field E^S. This is done as an integral of spherical waves, as written in Eq. 4, resulting from field E^S[4]:

$$E_M^O(\mathbf{r}_0) = \iint \frac{1}{i\lambda} E_A^S(\mathbf{r}) \frac{e^{ikR(\mathbf{r})}}{R(\mathbf{r})} dS. \tag{10}$$

Here we have made an integral of all spherical waves caused by E_A^S. $R(\mathbf{r})$ is the distance between $\mathbf{r}_0$ and $\mathbf{r}$. To simplify we will assume that the SLM device sits on the xy-plane at $z = 0$. We can rewrite E^S as incident electric field $E_i^S(x,y)$ along with the modulation

$$E^S(\mathbf{r}) = A^S(x,y) e^{i\phi^S(x,y)} E_i^S(x,y). \tag{11}$$

Here we have used $A^S(x,y)$ to describe the amplitude modulation function and $\phi^S(x,y)$ to describe phase modulation. We can also define $A^S(x,y)$ such that it is equal to 0 outside the SLM device, allowing us to use infinite boundaries on the integrals. Using paraxial approximation R can be written as:

$$R = \sqrt{(x-x')^2 + (y-y')^2 + z^2} \approx R_0 - \frac{xx' + yy'}{R_0}. \tag{12}$$

Here we have defined $R_0^2 = x^2 + y^2 + z^2$. Equation for $E^O(x,y,z)$ can now be rewritten as:

$$E^O(x,y,z) = \frac{1}{i\lambda} \frac{e^{ikR_0}}{R_0} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A^S(x',y') e^{i\phi^S(x',y')} E_i^S(x',y') e^{-ikxx'/R_0} e^{-ikyy'/R_0} dx' dy'. \tag{13}$$

The image produced by SLM is a Fourier transform $x', y' \to k_x, k_y$ of the modulated incident electric field, with $k_x = kx/R_0$ and $k_y = ky/R_0$. Based on this, an inverse Fourier transform can be used to find the amplitude and the phase modulation $A^S(x,y)e^{i\phi^S(x,y)}$ from the desired electrical field.

2.4.1 The phase retrieval problem

We have seen that the potential field is proportional to the light intensity and that the electric field on the SLM plane is an inverse Fourier transform of the electric field in the optical trap plane. We also know that the intensity of light can be expressed as $I^O \propto |\mathbf{E^O}|^2$. This can be used to express the electric field as:

$$E^O(x,y) \propto \sqrt{I^O(x,y)}e^{i\theta^O(x,y)}. \tag{14}$$

In this context, the electric field is represented as a scalar value because it is assumed that the SLM process does not affect the polarization of the field. However, a coordinate-dependent phase $\theta^O(x,y)$ is introduced, which can be set arbitrarily without affecting the resulting intensities I^O while changing the desired electric field in the SLM plane E^S.

The phase $\theta^O(x,y)$ can be used to compensate for the limitations of the SLM device by finding a new phase that creates an intensity field closer to the desired one. There are several algorithms available for phase computation, such as the Gerchberg-Saxton [5] algorithm or the MRAF algorithm [6].

The Gerchberg-Saxton algorithm and the MRAF algorithm are used to compute the phase $\theta^O(x,y)$ by iteratively updating the modulation of the electric field until the desired intensity is achieved. These algorithms can be used to optimize the performance of the SLM device by compensating for its limitations and producing more accurate intensity distributions.

3 Spatial light modulators

Spatial light modulators are digitally-controlled devices used to manipulate the properties of light, such as its amplitude, phase, and polarization, with high spatial resolution. They consist of an array of tiny pixels that can be individually controlled to produce complex light patterns. There are two options for spatial light modulators (SLMs): those using liquid crystals (LC-SLM) and those using digital micromirror devices (DMD). Both types of SLMs have their own benefits and drawbacks. One limitation of both LC-SLMs and DMDs is that they are digitally controlled, which can limit the precision of the modulation. The digital nature is seen both in discrete modulation steps as well as the physical modulator, since they are made of small modulation elements, often referred to as pixels. Furthermore this commonly reflects on the whole working procedure since it is beneficial to describe the desired intensity as an array of pixels. The dimensions of the intensity pixel array are often selected based on the properties of the SLM device. Additionally, neither a single DMD nor LC-SLM can independently modulate both phase and amplitude at the same time. Multiple SLM systems exist that can independently modulate both phase and amplitude [7] , but they are complex and intricate. It is important to note that these systems require careful calibration and alignment to ensure optimal performance.

3.1 Digital micromirror device

A digital micromirror device (DMD) is a type of spatial light modulator that uses an array of tiny mirrors to control the direction of reflected light as shown in Figure 5. Each mirror can be individually tilted to reflect light either towards or away from the projection lens. This is a significant drawback of DMDs, since the modulation is limited to two levels, either on or off. Yet they are very fast, up to around 20kHz, and can be used to rapidly switch between different light patterns, making them useful for applications that require high-speed modulation of light [8].

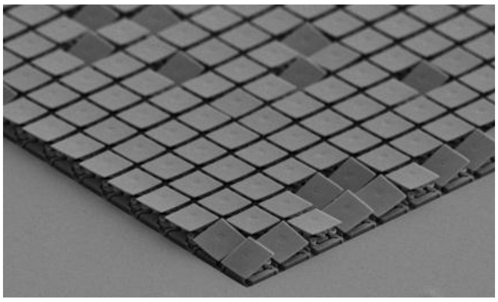

Figure 5: A scanning electron microscope image of a DMD. The angle of individual mirrors can be controlled digitally, which can be seen on the image. Image from Ref. [9].

3.2 Liquid crystal spatial modulators

A liquid crystal spatial light modulator (LC-SLM) is a type of SLM that uses a layer of liquid crystal material to modulate the properties of light. The liquid crystal layer is sandwiched between two glass plates and is divided into a grid of tiny pixels, each of which can be individually controlled.

The LC-SLM works by applying an electric field to the liquid crystal material, which changes the orientation of the liquid crystal molecules and alters the properties of the light passing through as can be seen on Figure 6. By varying the electric field applied to each pixel, the LC-SLM can be used to create complex light patterns with high spatial resolution.

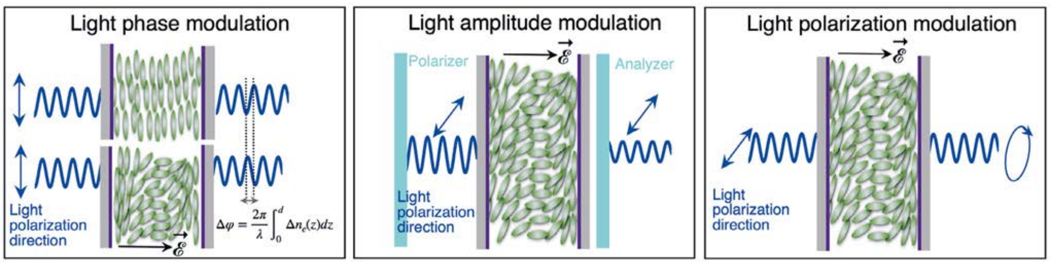

Figure 6: LC-SLM allow modulation of light phase, amplitude or polarization, which is determined by different configurations of liquid crystals. The liquid crystal material is located between two transparent electrodes which generate an electric field. This consequently changes the behaviour of liquid crystal material between the electrodes which modulates the incident light. Image from Ref. [10].

4 SLM system

It is beneficial to describe the use of an SLM system inside by considering a specific example. Regardless of whether an LC-SLM or a DMD device is used, the whole optical system is in general almost the same. The differences can be seen in the algorithms that create the modulation patterns. Hence we can take a look at a high-performance system used to create optical lattices, etc. described in Ref. [8]. The setup uses a DMD device due to its ability for fast modulation. As can be seen in Figure 7, this system, like many others, can be divided into several distinct parts.

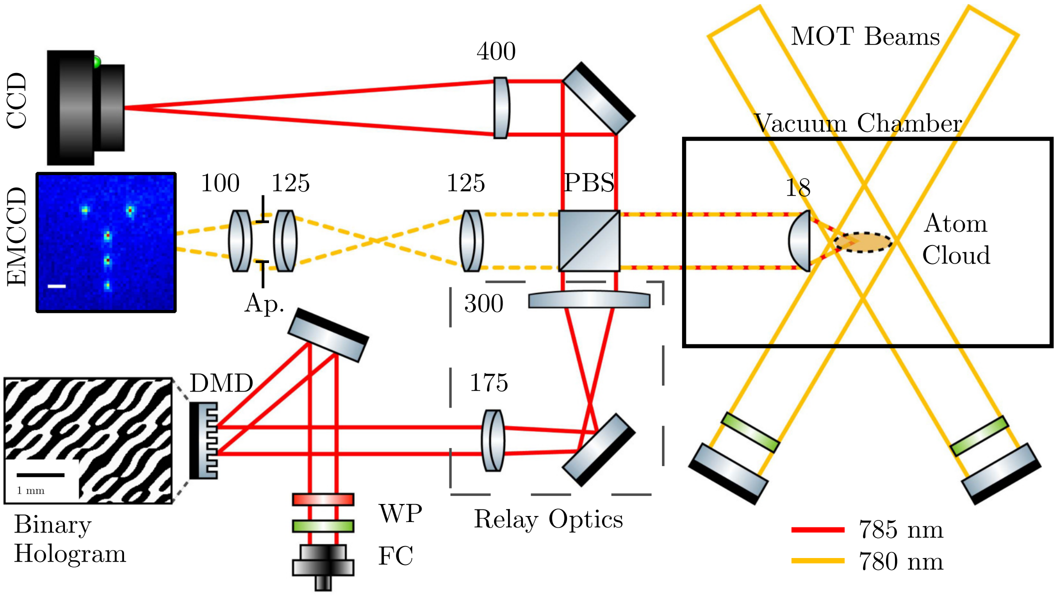

Figure 7: An example system for optical trapping of rubidium-87 using a DMD. Light comes from a fiber collimator (FC). Polarization of light is then prepared using a wave plate (WP), after which it is directed on to a DMD. Light exiting the DMD is then routed through relay optics, which bring the light to the desired location as well as provide the correct magnification. Using a polarizing beam splitter (PBS), the light can then be routed to the trap, as well as to the CCD camera used for feedback. Yellow lines show the 780 nm light used to create the magneto-optical trap (MOT) along with imaging of the cloud. Image from Ref. [8].

The first part involves preparing a beam of light with waveplates, polarizers, and lenses to expand the beam so that it covers the SLM device adequately. Ideally the beam impinging on the SLM device should be well collimated, which will reduce variation of phase and will illuminate the SLM area as evenly as possible. This is necessary to achieve maximum possible resolution. Some LC-SLM may also require proper polarization on the input side, which requires additional polarization optics.

The second part is the SLM module, where different types of optics can be used to separate the incident and refracted beams. The type of SLM used will determine the necessary optics.

The third part involves relay optics, which use lenses and mirrors to resize the output image to the correct size of the trap and route the light to the correct place.

Finally the light pattern is measured. The results are used for diagnostics and provide feedback to the phase retrieval algorithm.

4.1 Example uses of box trap potentials

Finally we can quickly take a look at why it is so beneficial to have the ability to create a box potential. Many experiments showcase these benefits (e.g. Ref [1]). For the sake of simplicity, we can look at the measurement of sound waves along a cloud of ultracold atoms inside a box potential, examples of which can be seen in Figure 8. This has been used to study superfluidic phase transitions. Here the argument for a box potential is clear since it ensures uniform density of the atoms, resulting in a uniform wave. In contrast, a harmonic potential where both density of atoms and potential are not uniform would make the acoustic waves much harder to observe since the speed of sound is dependent on the density of atoms.

Studies of sound waves inside ultracold atom gasses can further be used to study superfluidics, etc. These form a subset of equilibrium experiments where the system is weakly perturbed This can then be used to perform spectroscopic and transport measurements. Furthermore experiments can be done with different levels of perturbation, which allow research of different phenomena [1]. For example, non-equilibrium experiments, where the system is perturbed in a much stronger fashion, can show formation of different domains inside clouds of ultracold gases along with creation of vortices, etc.

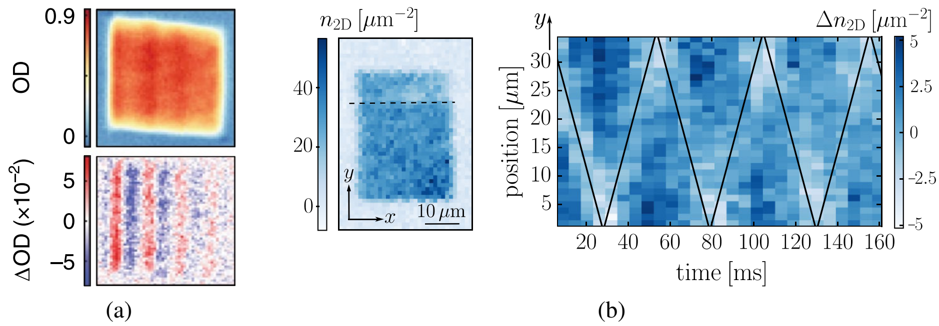

Figure 8: (a) Propagation of sound waves in a Fermi gas. Bottom picture shows deviations from average optical density in the trap. Sound wave fronts can be clearly seen as vertical lines. Taken from [1]. (b) Sound wave propagation in Bose gas. Left: image of the gas inside the trap. The area above the dashed line was used to perturb the system. Right: variation of number density of atoms in regards of time and y coordinate. Every point depicts mean value of Δn_{2D} along x axis at a given y value. The dip is fitted with a triangle function depicted as a solid black line. Both images in (b) are taken from [11]

5 Conclusion and outlook

Spatial light modulation (SLM) is a powerful and flexible technology used in physics experiments to control and manipulate the behavior of atoms and other quantum systems. One of the advantages of SLM is that it allows for the creation of precise and complex optical potentials with arbitrary shapes, which can be used to explore a wide range of phenomena in atomic physics, quantum information processing, and other areas. The development of SLM technologies, such as digital micromirror devices and liquid crystal modulators, has made it possible to create high-resolution and affordable modules that can modulate light with high resolution and fast refresh rates. One example of the application of SLM in physics is the creation of box traps, which provide a potential with a flat bottom that enables easier creation and interpretation of experiments. Additionally, SLMs allow creation of potentials that are time dependent, which offers the ability to study time evolutions of various quantum systems. With further advancements in SLM technologies, it is possible to explore other fields such as atomtronics and quantum computing [12].

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