Univerza *v Ljubljani*
Fakulteta za *matematiko in fiziko*

Seminar I

Bose Fireworks

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Abstract

Bose fireworks are matterwave jets that are emitted from a Bose-Einstein condensate when interaction between atoms is modulated. We describe their discovery and discuss stimulated atom collisions that are responsible for jet emission. In the second part of the seminar we present the Gross-Pitaevski equation and its use for numerical simulations of jet dynamics. Furthermore, we present the interaction Hamiltonian and the use of Bogoliubov approximation for calculating the number of atoms in the modes. To conclude, the potential use of jets for quantum simulations and metrology is discussed.

Contents

1 Introduction

Bose-Einstein condensate (BEC) is a state of matter that appears when a dilute gas of bosons is cooled to very low temperatures. The defining quality of bosons is that in contrast to fermions, there is no exclusion principle and therefore any number of bosons can occupy the same quantum state. When the temperature reaches the critical temperature, all bosons occupy the ground state and all atoms can be described using a single macroscopic wave function.

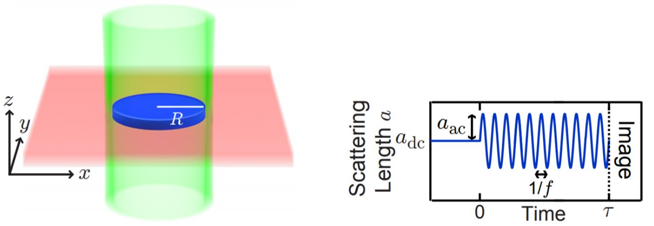
These phenomena were first predicted in a paper by Bose and Einstein that was published in 1924 [1]. Preparing the BEC is difficult because it requires ultrahigh vacuum and micro to nanokelvin temperatures. Since the 1970s, many atom cooling and trapping methods were developed, which paved the path to BEC. In modern experiments the atoms are first slowed down with laser cooling [2], and caught into a magneto-optical trap. Then evaporation in a magnetic or dipole trap is used to increase phase-space density and cool the atoms to nanokelvin temperatures. The Nobel prize in Physics 2001 was awarded to Eric A. Cornell, Wolfgang Ketterle and Carl E. Wieman for the very first experimental conformation of BEC with rubidium atoms in 1995 [3].

The research of the condensate is very intriguing because it offers a rare insight into the quantum nature of the world. The density of the condensate is directly related to its wavefunction, and it can be measured due to its macroscopic scale. The system is also easily tunable by laser technology and magnetic fields [4]. Laser light exerts a dipole force on the atoms which can be attractive or repulsive, depending on its wavelength. Moreover, the interaction between atoms can be changed through Feshbach resonances by applying a magnetic field [5]. All the above-mentioned properties of this so-called "fifth state of matter" make it an extraordinary plattform for simulating complex systems such as the dynamics of electrons in periodic lattices [6] and emission of Hawking radiation [7]. It can also be used to make atom lasers, atomic clocks and quantum gravitational, rotational or magnetic sensors [8] or even as a platform for quantum computing [9].

In this seminar, we will present a surprising phenomenon, discovered in 2017 by the Chicago Ultracold atomic and molecular physics group [10]. When modulating the interaction between atoms in a condensate, emission of matter-wave jets was observed. Because of its form and radial spread from the condensate, the emitted jets were named Bose fireworks. Some experimental results in two and one-dimensional geometry will be presented along with the fundamental features of the phenomena. In the second section, we will discuss the ways to describe this effect numerically and analytically. We will present the results of numerical simulations of the Gross-Pitaevskii equation [11] and see how the Bogoliubov approximation can be applied to this problem [12]. In the end, we will draw some parallels to other physical phenomena and discuss the use of Bose fireworks for quantum simulations and metrology.

2 Bose fireworks

Bose fireworks are matterwave jets, emitted from BEC as a result of modulation of the interaction between atoms in condensate [10]. The frequency of the modulation is in the kHz range. In order to understand where the name comes from, let us look at the first-ever published images of the jets. Using lasers and magnetic field, a condensate of caesium atoms is confined into a thin disk with radius $8.5\,\mu$m and height $0.5\,\mu$m as shown in Fig. 1(a). The vertical dimension is much



(a) Optical trap for BEC. (b) Modulation of interaction.

Figure 1: Experimental setup for producing Bose fireworks. Adapted from [10].

smaller than the horizontal one, which makes the setup quasi-two-dimensional. The magnetic field is modulated near a Feshbach resonance at 1.7 mT, resulting in periodic change of the interaction between atoms in the condensate. The interaction can be characterized by the scattering length, which is positive for repulsive interactions and negative for attractive ones and is shown in Fig. 1(b). In this case, the offset a_{dc} is small, so the scattering length oscillates between positive and negative values. After a couple of milliseconds jets of atoms are emitted and propagate in the radial direction, away from the central condensate. Here it is important to note that all emission takes place in the plane of the condensate. Fig. 2 shows the jets at different times after the onset of modulation. Pictures from the experiment resemble fireworks and that is why these matter-wave jets are called Bose fireworks.

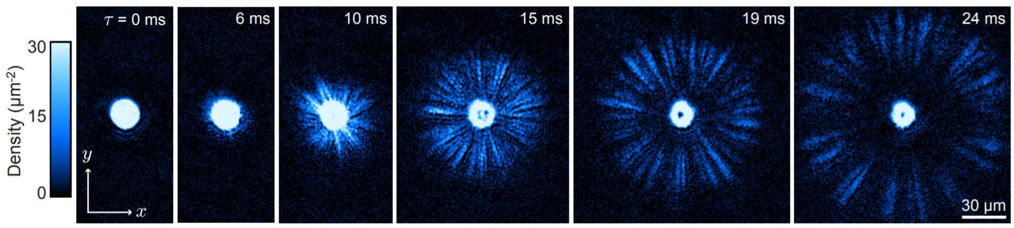


Figure 2: First-ever published photo of the time evolution of Bose fireworks. The jets appear a few milliseconds after the onset of modulation and propagate in the radial direction in the plane of the condensate. Adapted from [10].

It was soon noticed that the jets only appear when the modulation amplitude exceeds a threshold. The threshold amplitude shows a square root dependence on the modulation frequency [10].

The emission of Bose fireworks exhibits spontaneous symmetry breaking. The direction of the jets is random and changes in each repetition of the experiment. However, the researchers noticed that in some pictures, bright jets appear at opposite sides of the condensate. This prompted the study of the angular correlation function

$$g^{(2)}(\phi) = \frac{\langle \int d\theta n(\theta)\left[n(\theta+\phi) - \delta(\phi)\right]\rangle}{\langle \int d\theta n(\theta)\rangle^2}, \tag{1}$$

where $n(\theta)$ is the angular density of the atoms and angular brackets represent an ensemble average over many images. This functions shows the degree of correlation between atom densities in two directions as a function of the relative angle ϕ. The results for several different modulation durations are shown in Fig. 3. We see that there are high correlations at small angles. From the peak at $\phi = 0°$ the angular width of one mode (one jet) was determined to be 3°. There is another peak at $\phi = 180°$ which indicates that two atoms with opposing momenta are created.

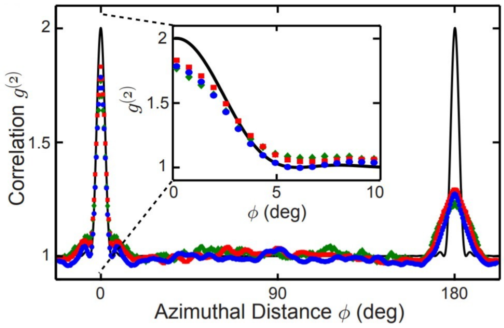
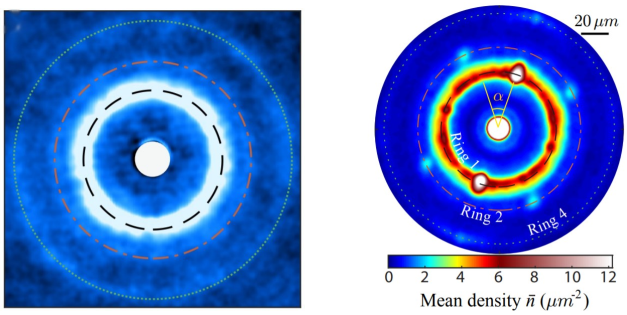


Figure 3: Angular correlation function of the jets. Adapted from [10].

It is interesting that the direction of the jets changes in every run of the experiment. In fact, if one averages out the pictures from 209 repetitions of the experiment, one gets several uniform rings (Fig. 4(a)) around the central position of the condensate [12].



(a) Mean image. (b) Extracted pattern.

Figure 4: Mean image and hidden turtle pattern of Bose fireworks. Adapted from [12].

However, using machine learning, a pattern resembling a turtle (Fig. 4(b)) has been found by rotating pictures in a way that maximized the angular variance of the mean image. We see distinct first-order jets in opposite directions and 8 second-order jets. In Fig. 5, we can see microscopic processes that account for this pattern. The primary collisions (Fig. 5 (a)) are collisions of two atoms in the condensate that split the energy quanta $\hbar\omega$ from the modulation and gain the opposite momentum $\hbar k_f$. This process is the most common and consequently, two bright white spots can be seen on opposite sides of the ring. Moreover, two types of secondary collisions can be recognized. One is a collision between an atom in the condensate and an atom in first-order

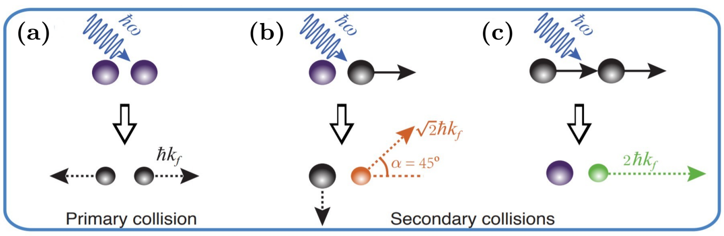


Figure 5: Stimulated two-atom collisions. Adapted from [12].

jet (Fig. 5 (b)), where one of the atoms propagates at an angle of 45° with respect to the first order jets. Four such spots can be seen on the turtle pattern. The less-costrasting dots at an angle 90° from the first order jets are also a product of this type of collision. Lastly, another two spots in the direction of first-order jets can be observed in the image. These correspond to collisions between two atoms in the first order jets (Fig. 5 (c)). After the interaction, one atom remains static in the central condensate and the other one propagates with a speed twice as large as the first order jets. This is why the second spot is further away from the central condensate than the first-order jets. There are more than 600 tertiary jets in this two-dimensional case, so they have not been mapped out.

As we have already mentioned, the shape of the jets is determined by the geometry of the condensate. The results that were discussed above were produced on a two-dimensional BEC in disk-like shape. However, at the Cold atom laboratory at Jožef Stefan institute, the BEC is prepaired in **quasi-one-dimensional geometry**. This is achieved by prepairing the condensate in a crossed trap and then turning one of the beams off. The atoms can move along this one-dimensional channel. By tuning the interaction to slightly negative, the condensate is prepared as a soliton, that is a nondispersing wave packet [13]. That makes it self confined. The simplicity of this geometry makes it very convenient for jet formation experiments since it makes jet categorisation easier [11]. We don't need to account for angular variations, as all BEC, jets included, are confined in a one-dimensional channel created by a laser beam as shown in Fig. 6. In this geometry, a millisecond after the modulation begins, two first-order jets (J1) are emitted from the condensate and travel along the channel in opposite directions. After that, second-order jets (J2) which are twice as fast appear. The jet speeds are usually a couple of millimeters per second, depending on the modulation frequency.

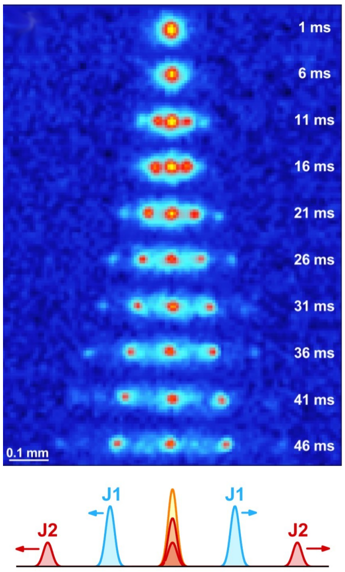


Figure 6: First and second-order Bose fireworks, propagating along the channel in a quasi-one-dimensional setup. Adapted from [11].

As we can see from the graph in Fig. 7, which shows how the number of atoms in first and second-order jets increases with modulation time, the second-order jets form after the first-order jets. This goes in line with the explanation that a second-order jet appears because of the interaction between two atoms in a first-order jet. We notice that after the initial exponential growth, the number of atoms in a jet saturates. That happens because the condensate is depleted. The number of atoms decreases, fewer stimulated collisions occur, and additional pairs of atoms do not form. In principle, higher-order jets would form if there were more atoms in the condensate (there are usually about 10 000 atoms in the condensate at the beginning). Another limiting factor

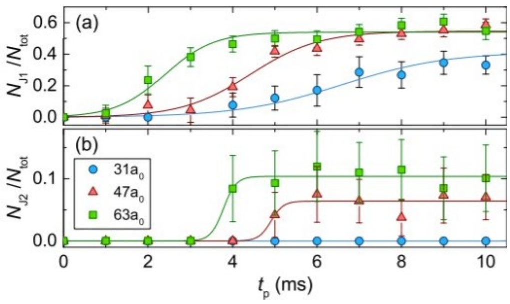


Figure 7: The number of atoms in (a) first and (b) second-order jets as a function of modulation time. Adapted from [11].

is the signal-to-noise ratio. As we can see from the picture above the higher-order jets are less prominent and could blend into the noisy background.

3 Theoretical description

Bose fireworks have not been predicted prior to their experimental observation. After observing their properties in experiments, the processes behind their formation were identified to be stimulated collisions of the atoms [10].

3.1 Gross-Pitaevskii equation

Bose-Einstein condensate is a many-body problem that can not be solved analytically. The dynamics of BEC can be described using the Gross-Pitaevskii equation (GPE) [15]. This mean-field approximation characterizes the ground state of a system of identical bosons. It is derived by assuming a low density of the gas and contact interaction between atoms. We set all particles in the ground state with the single-particle function $\psi(\mathbf{r}, t)$. Than we use variational method to minimize the energy. That condition determines the GPE

$$i\hbar\frac{\partial}{\partial t}\psi(\mathbf{r}, t) = \left(-\frac{\hbar^2}{2m}\nabla^2 + V_{ext} + g|\psi|^2\right)\psi, \tag{2}$$

where m is the mass of an atom in the condensate, $|\psi|^2$ is the atomic density, an external potential is described by V_{ext} and the coupling constant g can be derived from the scattering theory and is

$$g = \frac{4\pi\hbar^2}{m}a_s, \tag{3}$$

where a_s is the scattering length. The scattering length can be either positive for repulsive interactions or negative for attractive ones. To produce Bose fireworks, we modulate the magnetic field using a Feshbach resonance to periodically change the scattering length as

$$a_s(t) = a_{dc} + a_{ac}\sin\omega t. \tag{4}$$

The GPE is a nonlinear Schrödinger equation. Its mean-field nature can be seen if we define the potential energy factor as $V = V_{ext} + g|\psi|^2$. Because of its non-linearity, exact solutions can only be found for some specific cases such as a free particle and soliton [16]. Alternatively, it can be solved using numerical methods, which has been done for the above mentioned cases of two [14] and one-dimensional [11] condensate. In both instances, the simulations show that prior to jet formation, **density waves** appear and amplify rapidly in the condensate. The emission process in one dimension is shown in Fig. 8 and can also be described by means of parametric resonance, where the density waves produce a grating that amplifies itself.

The density waves were confirmed experimentally as well, and it was shown that the jet structure is predetermined by density waves in the condensate [14]. Interestingly, the azimuthal density structure factor of the density waves pattern is equivalent to the real space emitted jet population $N(\varphi)$ observed after a long propagation time.

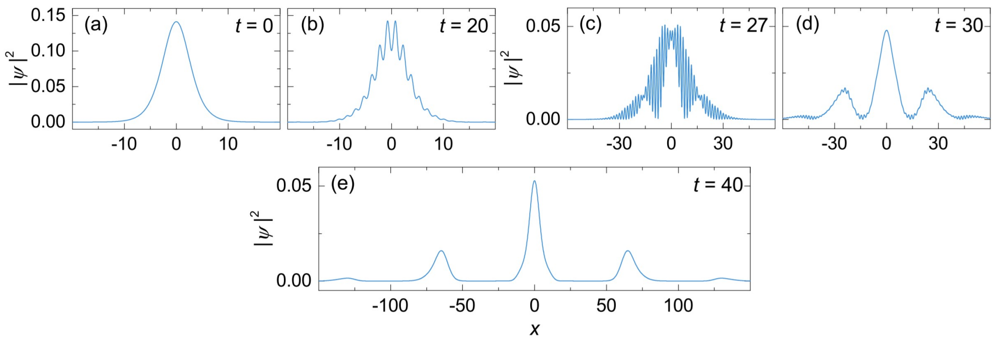


Figure 8: Numerical solutions of the GPE in one dimension. From (a) to (e), we follow the time evolution of the system. The position is denoted by x. Before the emission, density waves appear. Adapted from [11].

With this numerical simulation, we can correctly reproduce the dynamics of the system, including higher-order jets. The simulation shows good agreement with the experiment about the jet kinetic energies, and the thresholds we discussed earlier. However, this theory can't confirm one interesting quantum property of the jets that could be very useful for metrology, and that is entanglement.

3.2 Bogoliubov approximation

Another useful approximation for studying such systems is the Bogoliubov approximation. This approximation assumes that the number of atoms in the ground state is much larger than the sum of atoms in all excited states. In that case, an annihilation or creation of a particle in the ground state does not change the state, and the ladder operators become merely a numerical factor. This approximation is suitable for our system because as we can see from the experiment snapshots, the number of atoms in the condensate is much higher than the population of the jets. This approximation holds until the BEC becomes depleted, but at that point the jets do not form anymore.

When describing jet formation, the standard second-quantization Hamiltonian

$$H = \int dx \Psi^\dagger(x,t) \frac{p^2}{2m} \Psi(x,t) + \frac{g(t)}{2} \int dx \Psi^\dagger(x,t)\Psi^\dagger(x,t)\Psi(x,t)\Psi(x,t) + \frac{1}{\mu_0} \int dx |B(x,t)|^2 \quad (5)$$

can be simplified. The coupling constant g is defined same as above. In this case, the **interaction term** dominates the behaviour of the system. We write it in wave vector space [12], take into account energy and momentum conservation and arrive at the quantitative model that describes first-order jet formation

$$H_I = \hbar\tilde{\nu} \left[a_k^\dagger a_{-k}^\dagger a_0 a_0 + h.c. \right], \quad (6)$$

where $\tilde{\nu} = \frac{2\pi\hbar a_{ac}}{mV}$. The Hamiltonian describes how two stationary atoms in the condensate are annihilated and two atoms with opposite momenta k and $-k$ are emitted. The momentum is conserved. Using the Bogoliubov approximation we can substitute the annihilation operators of atoms in the central condensate with numbers $a_0, a_0^\dagger \to \sqrt{N_0}$, so the Hamiltonian can be written as

$$H_B = \hbar\tilde{\nu} N_0 \left[a_k^\dagger a_{-k}^\dagger + h.c. \right]. \quad (7)$$

Using the Heisenberg equation of motion, we can calculate the time evolution of the operators. We get a set of coupled differential equations

$$i\frac{da_k}{dt} = \tilde{\nu} N_0 a^\dagger_{-k} \tag{8}$$

$$i\frac{da_{-k}}{dt} = \tilde{\nu} N_0 a^\dagger_{k}, \tag{9}$$

which are solved by

$$a_k(t) = a_k(0)\cosh\gamma t - i a^\dagger_{-k}(0)\sinh\gamma t, \tag{10}$$

$$a^\dagger_{-k}(t) = a_{-k}(0)\cosh\gamma t + i a^\dagger_{k}(0)\sinh\gamma t, \tag{11}$$

where $\gamma = N_0\tilde{\nu}$. Assuming the states are not populated at $t = 0$, we can calculate that the number of atoms in first-order jets for short times rises as

$$n_k = \langle a^\dagger_k a_k\rangle = \sinh^2(N_0\tilde{\nu}t). \tag{12}$$

The same calculation can be done for the population of the second-order jets where the important terms in the Hamiltonian are

$$H_{II} = \hbar\tilde{\nu}\left[a^\dagger_{2k} a^\dagger_0 a_k a_k + h.c.\right] \tag{13}$$

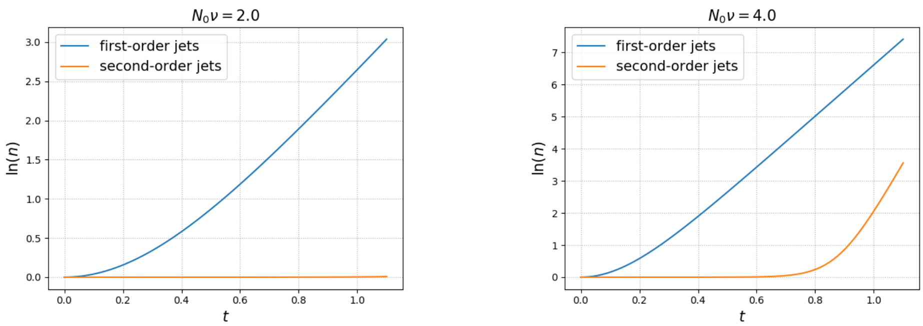
or

$$H_B = \hbar\tilde{\nu}\sqrt{N_0}\left[a^\dagger_{2k} a_k a_k + h.c.\right] \tag{14}$$

in the Bogoliubov approximation. The number of atoms in the second-order jet rises as

$$n_{2k} = \langle a^\dagger_{2k} a_{2k}\rangle = \frac{1}{8N_0}\left[\sinh(2N_0\tilde{\nu}t) - 2N_0\tilde{\nu}t\right]^2. \tag{15}$$

Plotting the results in Fig. 9, we see that this simple calculation shows that after a set period of modulation the first and second-order jets form if the number of atoms in the condensate is large enough (Fig. 9 (b)). However, if the number of atoms is too small, only jets of the first-order form because there are too few atom collisions (Fig. 9 (a)).



(a) (b)

Figure 9: The number of atoms in first and second-order jets as a function of time in the case of (a) low and (b) high number of atoms in the condensate.

4 Relation to other physical phenomena

We can try to understand Bose fireworks better by comparing them to other, more established physical phenomena. In principle, the jets appear because a scattering event that happens due to vacuum fluctuations stimulates additional scattering, resulting in collective scattering of many atoms. Moreover, the exponential amplification occurs above a certain threshold. This is similar to the light amplification that happens in a laser. The process can also be described by means of parametric resonance.

We can draw some parallels with nonlinear optics effects such as **spontaneous parametric down conversion** (SPDC). In this process, a pump photon with high energy is converted into a pair of photons with lower energy. Like in Bose fireworks production, energy and momentum are conserved. Moreover, just like fireworks in quasi-two-dimensional space, SPDC also exhibits a spontaneous symmetry break. The most important property of the SPDC output photons is that they are **entangled**, which is why they are used in most quantum communication schemes.

Because of the possible applications, current research is focused on proving jet entanglement. So far, it has been shown in Ref. [11] that the difference of the number of atoms in the left and right first-order jet exhibits a sub-Poissonian statistics, which is a precondition for jet entanglement. Jet entanglement would make precision metrology [17] below the shot noise limit possible. This is the fundamental restriction in interferometry with uncorrelated atoms [18], but it can be overcome with entangled photons. Furthermore, ensembles of entangled particles are required for quantum computing [19] and Bose fireworks are a promising platform for such applications because of the macroscopic atom numbers, which have not been achieved in other experimental realizations.

Bose fireworks have recently been reported to **spontaneously** form in spinor Bose-Einstein condensates, which makes them even more valuable for future applications. Spinor BEC has a spin internal degree of freedom [20]. Scientists have already been familiar with spin-changing collisions in such condensates, but it was reported in Ref. [21] that the initial collisions of atoms stimulate further collisions when they travel through the condensate. As a result, spin-correlated matter-wave jets are formed. The jets have different spin states and can be thought of as a macroscopic Einstein-Podolsky-Rosen state that is spatially separated. This state is very interesting for testing the fundamental principles of quantum mechanics.

Because of the good control over the trap geometry and interaction, BEC can also be used as a quantum simulator. In the case of Bose fireworks, **Unruh thermal radiation** can be studied. This effect describes how a Minkowski vacuum, seen by an inertial observer, corresponds to a thermal state of particles, seen by an accelerating observer [22]. Because extremely large accelerations are needed in order to achieve discernible temperatures, this effect has not been observed yet. However, it can be simulated with cold atoms. In fact, it was proposed in Ref. [23] that the frame transformation can be simulated as an evolution of a system with a Hamiltonian, which has the form of the jet formation Hamiltonian. That means we can understand the BEC as a vacuum state and jets as excitations. The emitted jets are the Unruh thermal radiation. In this quantum simulation, frequency and coupling constant are tuned. The results show that the emission has a thermal distribution and the results are in good agreement with Unruh's predictions.

5 Conclusion

This seminar focuses on the discovery of Bose fireworks and their properties. Stimulated two-atom collisions are discussed in order to describe microscopic events that produce the matter-wave jets. We present the use of numerical simulations to characterize the dynamics of the condensate as described by the Gross-Pitaevskii equation. Moreover, because of the characteristics of the system, we show that the Bogoliubov approximation can be used to reproduce the basic principles of jet formation. We draw some parallels to other physical phenomena and discuss the potential entanglement of the jets and their use as a quantum simulator of Unruh thermal radiation. To conclude, accidentally discovered Bose fireworks turned out to be interesting for fundamental research as well as applications in quantum metrology.

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