

Bose fireworks

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Seminar I

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Bose-Einstein
condensate

Feshbach resonances

Bose fireworks

Quasi-two-dimensional
geometry

Quasi-one-dimensional
geometry

Theoretical
description

Bogoliubov approximation

Gross-Pitaevskii equation

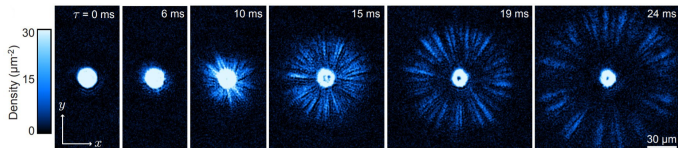
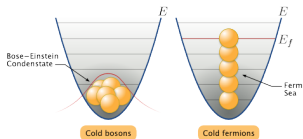


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Bose-Einstein condensate

- Bose-Einstein condensate (BEC) is a state of matter that appears when a dilute gas of bosons is cooled to very low temperatures.



[<https://culturacientifica.com/>]

- Insight into the quantum nature of the world.
- Easily tunable by laser technology and magnetic fields.

Bose-Einstein condensate

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Quasi-two-dimensional geometry

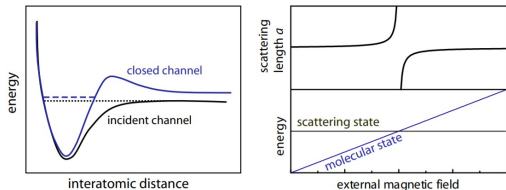
Quasi-one-dimensional geometry

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- Interaction potential \rightarrow scattering channel.
- Tune a bound molecular state in degeneracy with the scattering state of two free atoms.

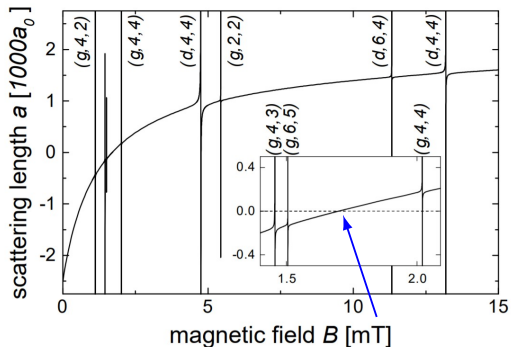


[M. Gustavsson, *A quantum gas with tunable interactions in an optical lattice*, Doctoral thesis (2008), Faculty of Mathematics, Computer Science and Physics, Innsbruck.]

- Scattering length diverges.

Feshbach resonances

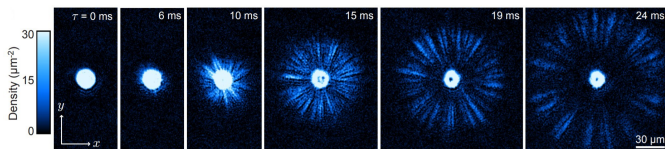
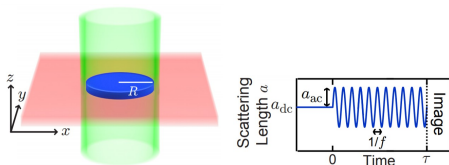
- Interaction between atoms \rightarrow scattering length.
- Scattering length can be tuned by a magnetic field.



[C. Chin et al., *Precision Feshbach spectroscopy of ultracold Cs₂*, Phys. Rev. A **70** (2017), 032701.]

Bose fireworks

- Bose fireworks are matterwave jets, emitted from BEC as a result of modulation of the interaction between atoms in condensate.

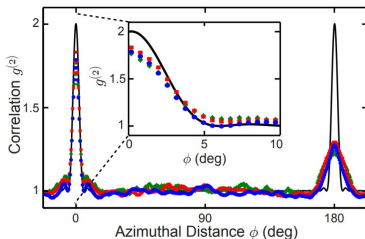


[L. Clark et al., *Collective emission of matter-wave jets from driven Bose-Einstein condensates*, Nature **551** (2017), 356–359.]

- Angular correlation function

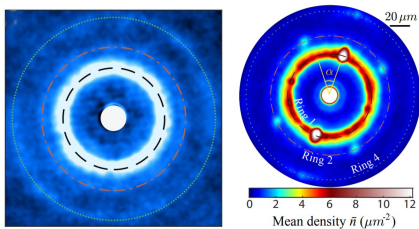
$$g^{(2)}(\phi) = \frac{\langle \int d\theta n(\theta) [n(\theta + \phi) - \delta(\phi)] \rangle}{\langle \int d\theta n(\theta) \rangle^2} \quad (1)$$

- Symmetry breaking.



[L. Clark et al., *Collective emission of matter-wave jets from driven Bose-Einstein condensates*, Nature **551** (2017), 356–359.]

- Machine learning, rotating pictures to maximize angular variance of the mean image.

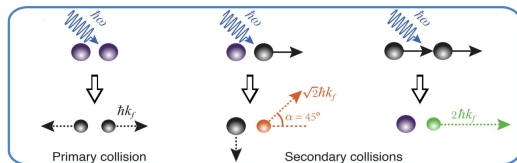
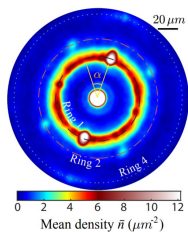


[L. Feng et al., *Complex correlations in high harmonic generation of matter-wave jets revealed by pattern recognition*, *Science* **362** (2019), 521-524.]

- Uniform rings \rightarrow turtle pattern.

Microscopic processes

- Stimulated two-atom collisions.



[L. Feng et al., *Complex correlations in high harmonic generation of matter-wave jets revealed by pattern recognition*, Science **362** (2019), 521-524.]

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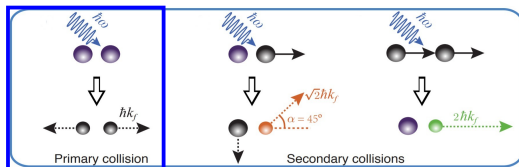
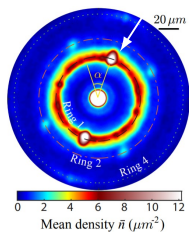
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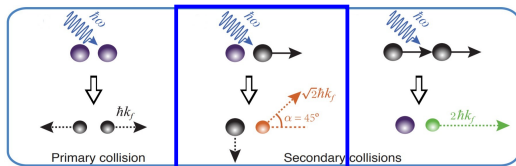
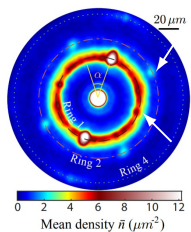
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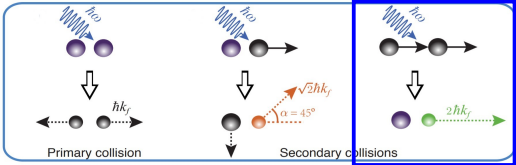
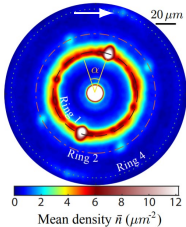
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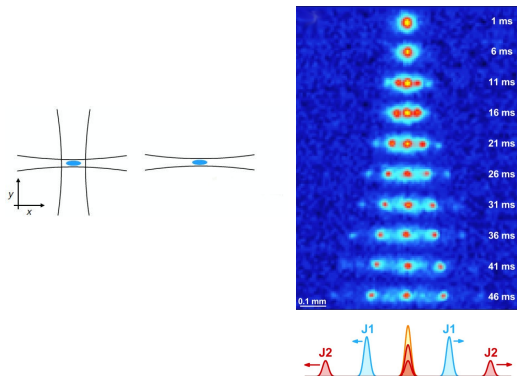
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Bose fireworks in quasi-one-dimensional geometry

- Simple geometry \rightarrow easier jet categorisation.



[T. Mežnaršič et al., *Emission of correlated jets from a driven matter-wave soliton in a quasi-one-dimensional geometry*, Phys. Rev. A **101** (3) (2020), 031601.]

- Number of atoms in first-order jets \rightarrow sub-Poissonian statistics \rightarrow entanglement.

Bogoliubov approximation

$$H = \int dx \Psi^\dagger \frac{p^2}{2m} \Psi + \frac{g}{2} \int dx \Psi^\dagger \Psi^\dagger \Psi \Psi + \frac{1}{\mu_0} \int dx |B|^2 \quad (2)$$

- Coupling constant $\rightarrow g = \frac{4\pi\hbar^2}{m} a_s$.

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- Coupling constant $\rightarrow g = \frac{4\pi\hbar^2}{m} a_s$.
- Interaction term, wave vector space.

$$\Psi(x, t) = \frac{1}{\sqrt{V}} \sum_k e^{ikx} a_k \quad (3)$$

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- First-order jets:

$$H_I = \hbar\tilde{\nu} \left[a_k^\dagger a_{-k}^\dagger a_0 a_0 + h.c. \right] \quad (4)$$

$$\tilde{\nu} = \frac{2\pi\hbar a_{ac}}{mV} \quad (5)$$

Bogoliubov approximation

- First-order jets:

$$H_I = \hbar\tilde{\nu} \left[a_k^\dagger a_{-k}^\dagger a_0 a_0 + h.c. \right] = \hbar\tilde{\nu} N_0 \left[a_k^\dagger a_{-k}^\dagger + h.c. \right] \quad (6)$$

Bogoliubov approximation: $a_0, a_0^\dagger \rightarrow \sqrt{N_0}$

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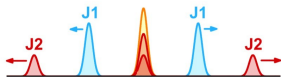
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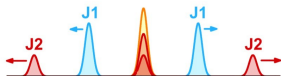
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Bogoliubov approximation: $a_0, a_0^\dagger \rightarrow \sqrt{N_0}$



- Second-order jets:

$$H_{II} = \hbar\tilde{\nu} \left[a_{2k}^\dagger a_0^\dagger a_k a_k + h.c. \right] = \hbar\tilde{\nu} \sqrt{N_0} \left[a_{2k}^\dagger a_k a_k + h.c. \right] \quad (7)$$

Gross-Pitaevskii equation

$$i\hbar \frac{\partial}{\partial t} \psi(x, t) = \left[-\frac{\hbar^2}{2m} \nabla^2 + V(x, t) \right] \psi(x, t), \quad (8)$$

- Mean-field approximation.

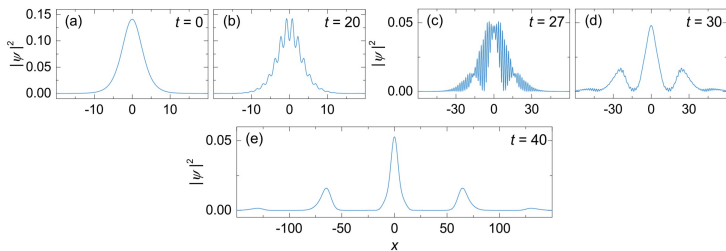
$$V(x, t) = V_{\text{ext}} + g|\psi|^2 \quad (9)$$

- Low density and contact interaction.

$$g = \frac{4\pi\hbar^2}{m} a_s \quad (10)$$

$$a_s(t) = a_{dc} + a_{ac} \sin \omega t. \quad (11)$$

- Density waves appear and amplify rapidly.



[T. Mežnaršič et al., *Emission of correlated jets from a driven matter-wave soliton in a quasi-one-dimensional geometry*, Phys. Rev. A **101** (3) (2020), 031601.]

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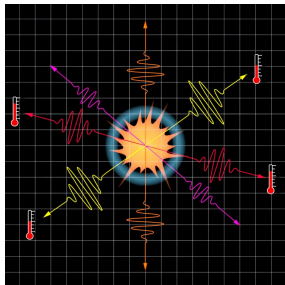
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Unruh thermal radiation

- Minkowski vacuum is seen as a thermal state by an accelerating observer
$$T_U = \frac{\hbar A}{2\pi c k_B}.$$
- Acceleration of $A = 2.5 \cdot 10^{14} \text{ m/s}^2$ for $T_U = 1 \mu\text{K}.$
- Rindler frame transformation can be simulated based on the evolution operator of a Hamiltonian of the first-order jet formation.
- Thermal distribution of the number of atoms in jets.
- Good agreement with Unruh's predictions.



[J. Hu et al., *Quantum Simulation of Coherent Hawking-Unruh Radiation*, Nat. Phys. **15** (8) (2019), 1745-2481.]

Conclusion

- New physical phenomena discovered by modulating the interaction between the atoms in BEC.
- Stimulated two-atom collisions.
- The jets are potentially entangled.
- Use as a simulator for Unruh thermal radiation.