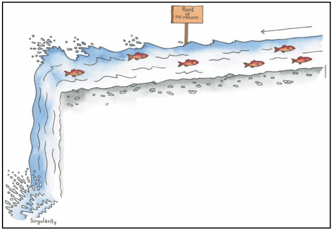
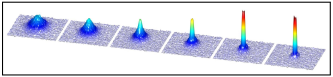
Modeliranje črnih lukenj z Bose-Einsteinovim kondenzatom

Matevž Jug

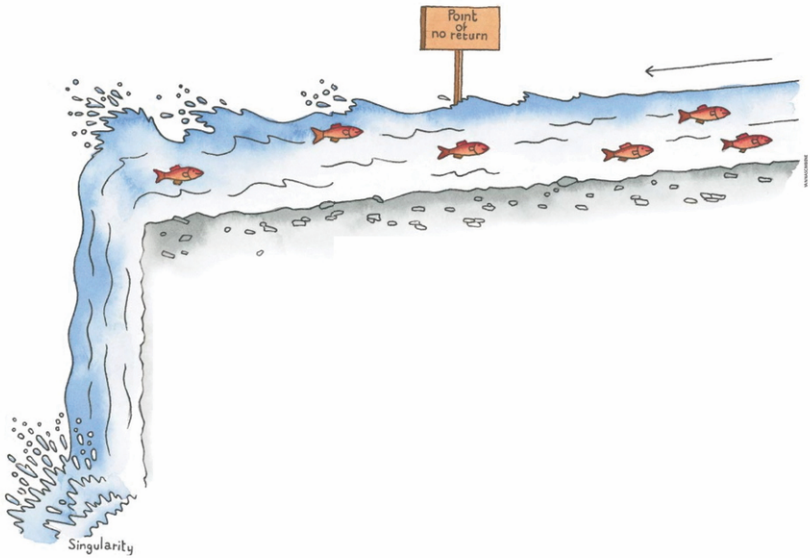
6. maj 2020

Uvod





Slap



Metrika

Minkowskega:

$$ds^2 = c_0^2 dt^2 - dr^2 - r^2 d\Omega^2$$

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$$\boxed{\frac{mc^2}{2} = \frac{GMm}{r_s}}$$

Schwarzschildova:

$$ds^2 = \left(1 - \frac{r_s}{r}\right) c_0^2 dt_s^2 - \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 - r^2 d\Omega^2$$

$$ds^2 = c_0^2 dt^2 - \left(dr + \sqrt{\frac{r_s}{r}} c_0 dt\right)^2 - r^2 d\Omega^2$$

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$$ds^2 = c^2 dt^2 - (dx - V(x) dt)^2$$

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V premikajočem se sredstvu:

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$$\Bigg\} \rightarrow V(r) = -c_0 \sqrt{\frac{r_s}{r}}$$

$$0 = c^2 \mathrm{d}t^2 - (\mathrm{d}x - V(x)\mathrm{d}t)^2$$

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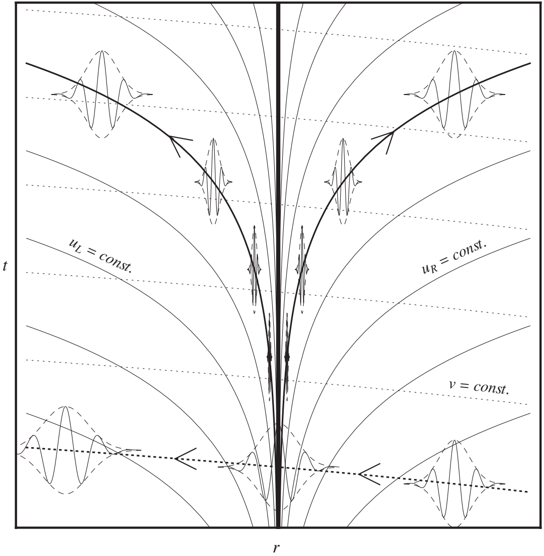
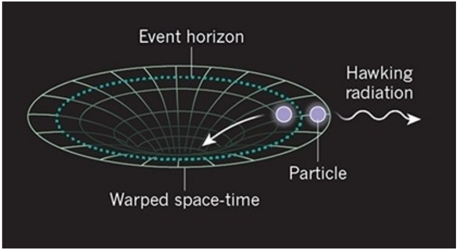


Hawkingovo sevanje



$$k_B T_H = \frac{\hbar g}{2\pi c_0} = \frac{\hbar c_0^3}{8\pi G M}$$

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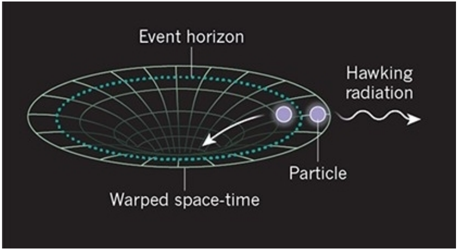


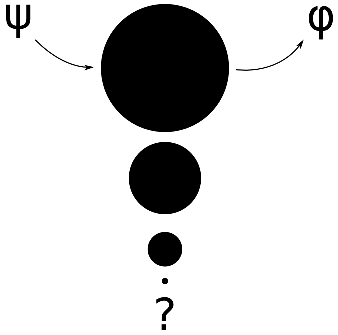
$$k_B T_H = \frac{\hbar g}{2\pi c_0} = \frac{\hbar c_0^3}{8\pi GM}$$

$$M = 2 \times 10^{30}\,\mathrm{kg} \rightarrow T_H = 60\,\mathrm{nK}$$

$$M = 75\,\mathrm{kg} \rightarrow T_H = 1.6 \times 10^{21}\,\mathrm{K}$$

Težave

Informacijski paradoks



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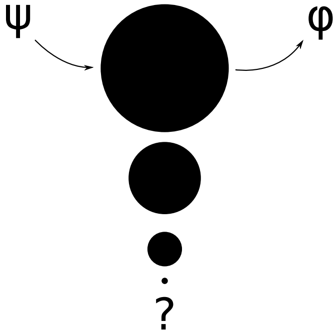
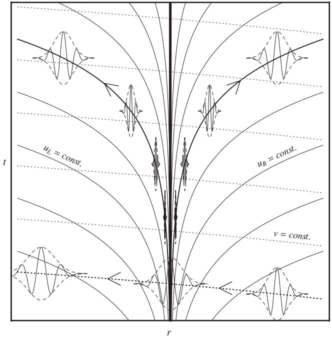
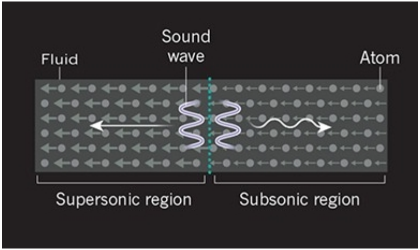


Transplankovski problem



Analogna gravitacija

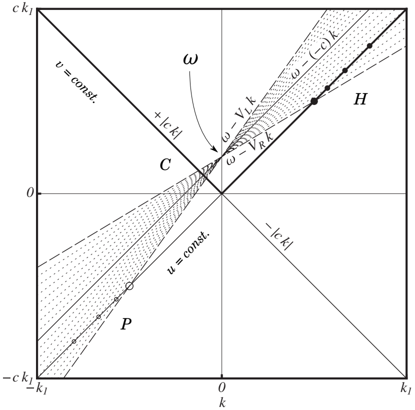


$$g = c\,\frac{\mathrm{d}}{\mathrm{d}x}(V + c)$$

$$k_B T_H = \frac{\hbar g}{2\pi c} = \frac{\hbar}{2\pi}\left(\frac{\mathrm{d}V}{\mathrm{d}x} + \frac{\mathrm{d}c}{\mathrm{d}x}\right)$$

Disperzija

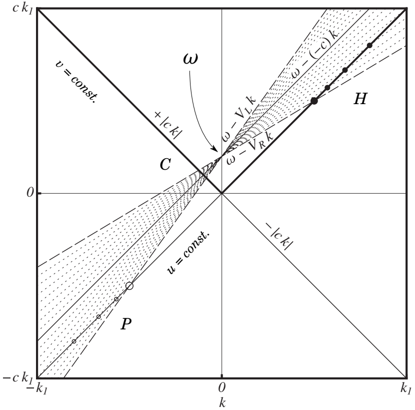
$$(\omega - Vk)^2 = c^2 k^2$$

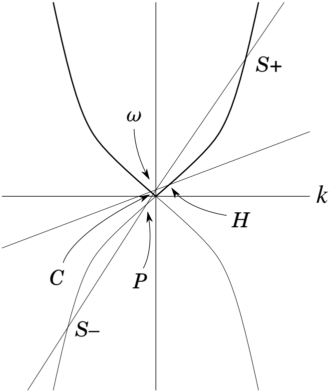


Disperzija

$$(\omega - Vk)^2 = c^2 k^2$$

$$(\omega - Vk)^2 = c^2 \left(k^2 + k^4/k_0^2\right)$$





Bose-Einsteinov kondenzat

De Broglieva valovna dolžina:

$$\lambda_B = \frac{h}{p} = \sqrt{\frac{2\pi\hbar}{mk_B T}}$$

$$T < 1\,\mu\mathrm{K}$$

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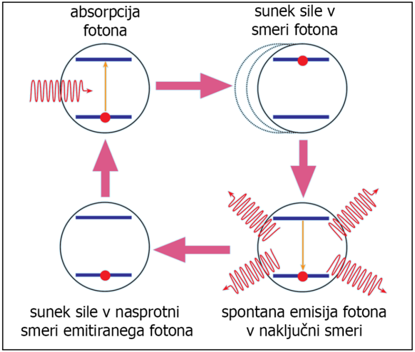
Sila laserskega žarka:



Ključne lastnosti

Disperzija Bogoljubova: $\omega^2 = \frac{\kappa n}{m} k^2 + \frac{\hbar^2}{4m^2} k^4$

- majhni k: $\omega^2 = \frac{\kappa n}{m} k^2 \quad \rightarrow \quad c = \sqrt{\frac{\kappa n}{m}} \approx 1\,\text{mm/s}$
- veliki k: $\hbar\omega = \frac{(\hbar k)^2}{2m} + \kappa n$

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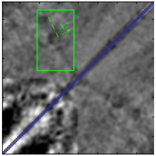
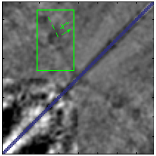
Koherenčna dolžina: $\kappa n = \frac{p^2}{2m} = \frac{\hbar^2}{2m\xi^2} \quad \rightarrow \quad \xi = \frac{\hbar}{\sqrt{2}mc} \approx 1\,\mu\mathrm{m}$

(meja za valovno dolžino fononov =
meja za glajenje porazdelitve gostote)

Korelacija gostote: 

Korelacija gostote:

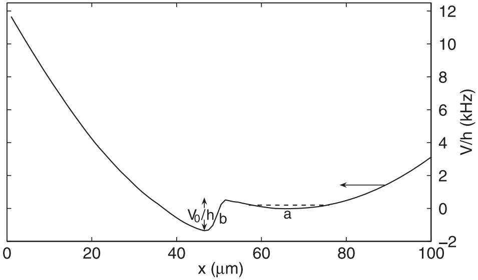


Ključne lastnosti:

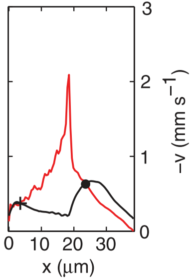
- kvantna narava $\rightarrow$ fluktuacije
- preprosta manipulacija
- nizka hitrost zvoka $\rightarrow$ doseganje nadzvočnega režima
- primerljivi temperaturi Hawkingovega sevanja in sredstva
- koherentnost $\rightarrow$ korelacijske meritve

Realizacija zvočne črne luknje

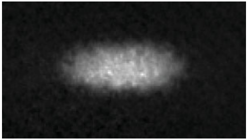
Skupen potencial



Hitrostna profila



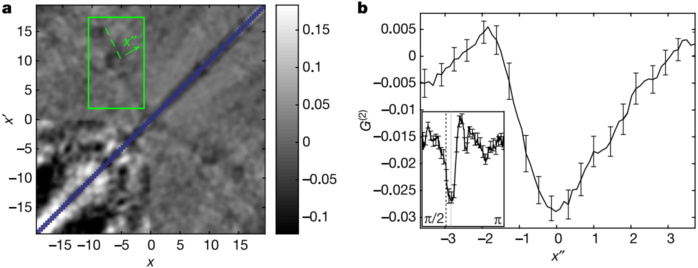
Fotografija kondenzata



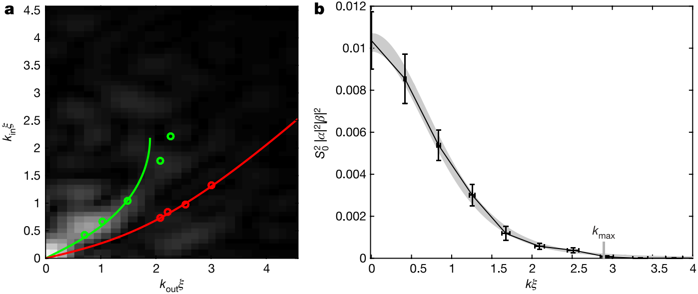
$$c = \sqrt{\frac{\kappa n}{m}}$$

$$V = -\frac{1}{n}\int_0^x \frac{\partial n}{\partial t}\,\mathrm{d}x'$$

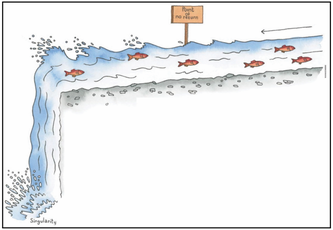
Termično Hawkingovo sevanje



Korelacijski vzorec	Integral vzorca
Korelacija valovnih vektorjev	Korelacijski spekter



$$k_B T_H = \frac{\hbar}{2\pi}\left(\frac{dV}{dx} + \frac{dc}{dx}\right) = 0.351\,\mathrm{nK}$$


Point of no return
Singularity

?

Viri slik

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