On the path to Bose-Einstein condensate

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Abstract

This seminar is intended as a brief introduction on techniques and mechanisms needed to achieve Bose-Einstein condensate (BEC). It also gives an intuitive explanation of what BEC is and of the concepts required to achieve it. The reader will, at the end, have a general overview over the knowledge needed for exploring this subject on his own.

1 Introduction

Superconductivity, superfluidity, and Bose-Einstein condensation (BEC) are among the most fascinating phenomena in nature. Their unusual and often surprising properties are a direct consequence of quantum mechanics as they are, in essence, macroscopic quantum phenomena.

Bose-Einstein condensate was first predicted in 1925 by Bose and Einstein and produced in 1995 by Eric Cornell and Carl Wieman using a gas of rubidium atoms. Upon reaching temperatures below one millionth of a degree above absolute zero, atoms' wave functions spread and start to overlap, resulting in strong correlations between particles - usually bosons. As a resoult, all the particles condense into a so-called "super atom" that behaves in perfect unison and can be described with a single wave function.[1]

Potential uses for such a state of matter include, but are not limited to:

- Simulation of condensed matter systems,
- Precision measurement (atomic clocks),
- Quantum computing,
- Quantum Optics,
- Quantum chemistry.

The indistinguishability of atomic particles becomes important when de Broglie wavelength becomes comparable to the distance between two neighboring particles. Since de Broglie wavelength is given by equation

$$\lambda_{\mathrm{dB}} = \sqrt{\frac{2\pi\hbar^2}{mk_BT}},$$

we can estimate the required temperature. Precise calculations give for a medium of particle density n and mean distance between particles $n^{-1/3}$ the critical temperature for BEC [2]:

$$T_c = \left(\frac{n}{\zeta(3/2)}\right)^{3/2} \frac{2\pi\hbar}{mk_B} \approx 3.31\frac{\hbar^2 n^{2/3}}{mk_B},$$

where ζ is the Riemann Zeta function $\zeta(3/2) \approx 2.6124$ and n is particle density. E.g. for gaseous alkali atoms at density $n = 10^{14}\mathrm{cm}^{-3}$ [3]

$$T_c \approx 0.17\ \mu\mathrm{K}.$$

Reaching so low temperature represents great experimental challenge, and usually requires multiple steps in the cooling process:

- Slowing an atomic beam,
- Optical molasses technique,
- The magneto-optical trap,
- Evaporative cooling.

The next sections will take a deeper look into each of these methods.

2 Slowing an atomic beam

In a seminar work published in the *Physical Review* in 1923 Compton explained the X-ray shift by attributing particle-like momentum to photons, an idea that was previously discussed by Maxwell and Einstein. This momentum is proportional to photon frequency and given by a well-known expression:

$$p = \frac{h\nu}{c} = \frac{h}{\lambda} = \hbar k.$$

Here p is the momentum, h is the Planck constant, $\hbar$ is the reduced Planck constant, ν is light frequency, λ is light wavelength and k is the angular wavenumber. Now let us consider a number of perfectly stationary atoms and a laser beam, at a frequency of atomic resonance, directed through the atoms. Atomic resonance is achieved when the energy of a single photon is precisely equal to a transition between two energy states of an atom. Due to the law of conservation of momentum, absorbed photons impart their momentum upon newly exited atoms, giving them a "kick" of recoil velocity $\mathbf{v}_{r1} = \frac{\mathbf{p}_{\text{photon}}}{m_{\text{atom}}}$. Upon returning to the ground state atoms emit another photon of same wavelength and changing it's velocity for $\mathbf{v}_{r2}$, but this time in a random direction. Considering a large number of emissions the average emitting recoil velocity $< \mathbf{v_{r2}} >= 0$. However the first kick always has the same direction, since all the photons come from the same light beam. This way it is possible to successfully accelerate a group of atoms. [5]

The same principle applies when slowing an atomic beam of fast atoms. Let us consider an atomic beam with sodium atoms with mass number $M = 23$ u and mean atomic velocity $v = 1000$ m/s. That would be the thermal velocity of Na atoms heated to 900 K We illuminate the atomic beam with a laser of $\lambda = 589$ nm, corresponding to the excitation of an electron from 3p to 3s orbital. Each photon with this wavelength carries about $p_{\text{photon}} = 1.1 \cdot 10^{-27} \frac{\text{kgm}}{\text{s}}$, and therefore $\mathbf{v}_{r1} = 3$ cm/s. To ideally stop sodium atoms we therefore need approximately 21000 photons, which can be done fairly easily with a laser beam of intensity $I = 6$ mW/cm^2. More precisely, an atom can be stopped on a distance of 1.1 m with mean deceleration of 10^6 m/s^2, which is a convenient distance for an experiment.

However, this concept is highly idealized. The calculation of the stopping distance assumes a constant deceleration, but for a given laser frequency atoms only experience a strong force over a narrow range of velocities for which the atoms have a range of Doppler shift approximately equal to the natural bandwidth of the laser (1 GHz for a typical He-Ne laser, although lasers for this experiments are usually frequency locked and have a much narrower bandwidth). With $v = 1000$ m/s doppler shift equals to about 1.67 GHz. This change must be compensated for in order to keep the force near its maximum for the whole slowing process.

Two pioneering laser cooling experiments used different methods to compensate for this change.

2.1 Solenoid slowing

William Philips and his co-workers sent the atomic beam through a tapered solenoid to make use of Zeeman effect. In a static magnetic field, different quantum states have a difference in energies

$$\Delta E = m_l \mu_B B,$$

where m_l is the z projection of angular momentum, μ_B is the Bohr magneton and B is magnetic field. The frequency shift caused by the Zeeman effect must therefore obey the condition:

$$\omega_0 + \frac{\mu_B B(z)}{\hbar} = \omega + kv,$$

where atomic resonance frequency ω_0, perturbed by Zeeman shift by $B(z)$ vertical component of the magnetic field, must equal laser frequency ω increased by the Doppler shift.

The magnetic field in a varying solenoid changes considerably with position and has the advantage of reducing the velocity of a large fraction of atoms inside the beam, since all atoms with velocities in range between v_{min} and v_{max} eventually interact with laser radiation and are swept along the slowing process. Here v_{max} is the speed at which the resonance condition applies where the field B is the strongest - ideally this is the speed at which atoms enter the solenoid. v_{min} is the speed at which atoms are in resonance at the lowest value of magnetic field B - ideally the speed of atoms when they exit the solenoid.

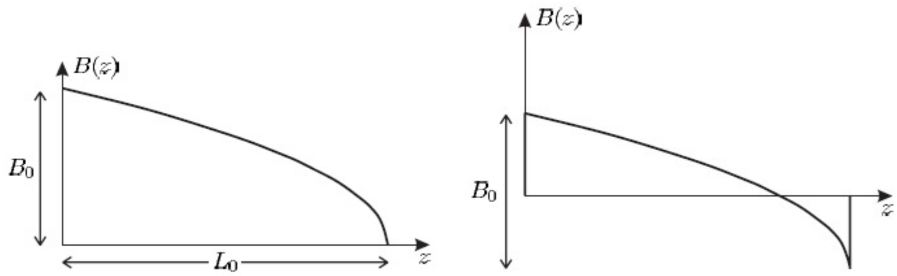


Figure 1: The original (left) and improved (right) magnetic field in a solenoid.

The magnetic field in the solenoid varies with the position as shown in Figure 1. Left diagram shows the field as it was used in the first Zeeman slowing experiment. Nowadays, some experiments use the variant on the right diagram, because while giving the same decrease in velocity, it presents certain advantages: the field has a lower absolute value so that the coils need less current-turns, it produces less field outside of the solenoid and the abrupt change in magnetic field allows the atoms a clean exit.

2.2 Chirp cooling

In 1976, Letokhov, Minogin and Pavlik suggested a general method of changing the frequency (chirping) of the cooling laser as to interact with all the atoms in a wide distribution and to stay in resonance with them as they are cooled. This concept was put into practice in 1985 by W. Ertmer, R. Blatt, J. Hall and M. Zhu. In this cooling method the frequency of the light must be swept over a range of more that 1 GHz in a few milliseconds. This is usually achieved through use of electro-optic modulators and radio-frequency techniques. However in practice, most experiments prefer the use of the above described solenoid cooling.

3 Optical molasses technique

In the previous section we have shown how we can slow down atoms in one direction. Since atoms in a gas move in all directions we need to apply cooling on all three orthogonal axis as well.

3.0.1 Concept

Imagine three pairs of slightly red-detuned lasers, i.e. below the resonance frequency, each lying on an axis of a cartesian system (Figure 2). For a stationary atom in the cross-section of all 6 beams the forces balance out. However for a moving atom the Doppler effect leads to an imbalance in the forces. When an atom moves towards a photon beam, Doppler shift causes laser frequency to move towards atomic resonance, resulting in a larger rate of absorbtion. This results in a slowing force for the atom. Expressed mathematically, this can be written as

$$F_{\text{molasse}} = -\beta v.$$

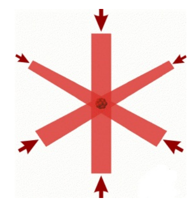


Figure 2: The optical molasses

It can be said that light exerts a damping force on the atom, just like a particle in a viscous fluid, for example molasses. Under optimal conditions the characteristic damping time is a few microseconds. This gives a speed limit for atoms entering the magneto optical trap, since faster atoms can simply escape the small volume of vacuum where all 6 laser beams intersect.

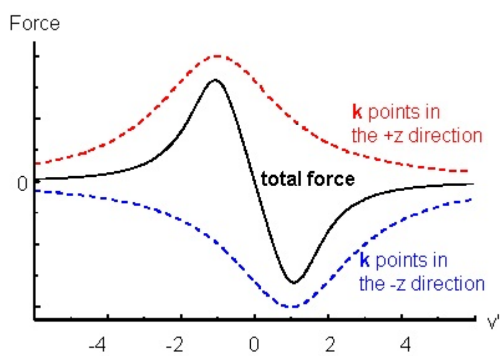


Figure 3: Damping force in relation to particle velocity. Red line represents the force of one laser beam, blue line the opposing force of the other laser beam. We make use of the almost linear part between the two extremes.

3.0.2 Doppler cooling limit

Let us take a look at how much can we cool atoms using this technique. Previously we have shown that absorbtion force F_{abs}, averaged in time equals a constant and emission force F_{emit} averages to 0. To understand the Doppler cooling limit, we need to take a look at the effect of fluctuation of these two processes. Spontaneous emission causes the atoms to recoil in random direction leading to a random walk of velocity. For a random walk of N steps, mean displacement equals to $\sqrt{N}$ times step length. If during a time t an atom scatters a mean number of photons N, the scattering rate R_{scatt} equals to

$$R_{\text{scatt}} = \frac{N}{t}.$$

Spontaneous emission causes the mean square velocity to increase along z axis as

$$\left\langle v_z^2 \right\rangle = \left\langle cos^2\theta \right\rangle v_r^2 R_{\text{scatt}} t,$$

which is greater than 0, since $\left\langle cos^2\theta \right\rangle$ averages to 1/3. Estimating the change in energy E_r given by a single photon and scattering rate R_{scatt} gives the mean square velocity spread in the six-beam optical molasses configuration as

$$\left\langle v_z^2 \right\rangle = 2E_r \frac{2R_{\text{scatt}}}{\beta},$$

where β is the damping factor [3]. Using the equation $\frac{1}{2}M < v_z^2 >= 1/2k_B T$ we can easily determine the minimum achievable temperature by such a technique. For sodium this predicted temperature equals about $T_D = 240\ \mu$K. However experimental measurements have found much lower temperatures under certain conditions. This is a rare example in which things turned out to be much better than expected. The theory explaining this phenomena is called Sisyphus cooling technique, which shall not be explained in this seminar. Details of this technique can be found in [3].

4 The magneto-optical trap

Now we shall tackle the issue of atoms escaping the optical molasses. Although the molasses can effectively slow atoms, it cannot by itself confine them in a small volume. This can be fixed with a correct choice of beam polarization and addition of a magnetic field gradient to the already red-detuned optical molasses. This system is called the magneto-optical trap or MOT.

Two coils, with currents in opposite directions, produce a magnetic quadrupole - the field has a zero intensity in the center between the coils, and increases linearly with distance from the center. Close to this point there is a uniform field gradient, that perturbs the atomic energy levels. If an atom moves away from the center, Zeeman shift causes an energy transition to shift closer to atomic resonance and increases the rate of absorbtion. Should we only use the optical molasses configuration from previous chapter, the rate of absorbtion would increase for both laser beams - the one pulling it towards the middle and the one pushing it away.

However, we know that circularly polarized photons carry one unit of angular momentum, an attribute, of which we take advantage in this situation. By adding a circular polarization to the laser beams only one transition for a laser beam becomes possible - for example $\Delta M_L = +1$ for right polarized light.

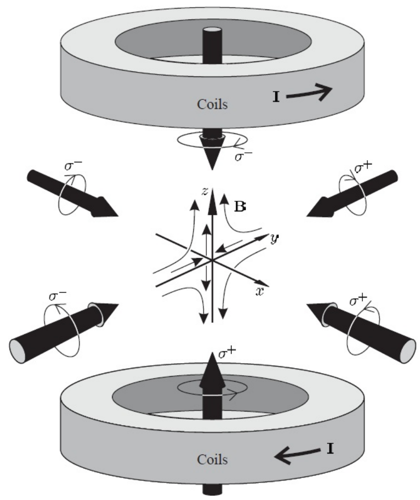


Figure 4: The magneto-optical trap.

Setting laser polarizations right, we increase the chances of absorbtion for the photons pushing the atoms back into center, resulting in a force proportional to the distance of the atom from the center. Combined with optical molasses technique, described in the previous section, the force light exerts on an atom can be described with equation:

$$F = -\alpha \mathbf{x} - \beta \mathbf{v}.$$

This apparatus is able to catch atoms with much higher initial velocities than optical molasses on its own and is used to collect atoms from a slowed atomic beam. When sufficient atoms have accumulated in the trap, the magnets are turned off, and the atoms are cooled by optical molasses alone. It turns out there are other, sub-Doppler cooling mechanisms which don't work as well under magnetic fields and the optical molasses technique on its own gives lower temperatures than a typical MOT.

5 Dipole trapping

In previous chapter we looked at the scattering force at which an object gains momentum as it absorbs or emits radiation. Another type of radiation force arises from diffraction of light, which is not in a wavelength of atomic spectrum and has a very low probability of absorbtion.

If a photon enters and then exits a prism, deflecting by an angle θ, the prism feels a push of force equal to $\hbar \cdot k \cdot 2 \cdot sin(\frac{\theta}{2})$ for a single photon or

$$F = \frac{IA}{c} \cdot 2 \cdot sin(\frac{\theta}{2}),$$

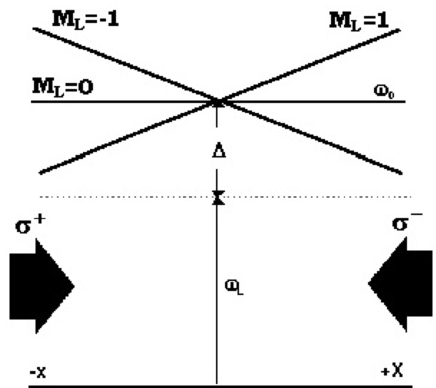


Figure 5: Zeeman shift causes the right transition energies to match with lasers'. Here M_L describes the state with vertical component of angular momentum as listed, ω_z the resonance frequency without Zeeman shift, and σ the direction of circular polarisation of the laser.

for a light beam of intensity I traveling through a cross-sectional area A (perpendicular to the direction of motion. $2 \cdot sin(\frac{\theta}{2})$ is the difference between the incoming and outgoing momentum of photons which increases with the refractive index of illuminated material.

Let us now consider a small glass sphere in a non-uniform laser beam with maximum intensity in the middle (let us declare this as axis X and decreasing as we move away - much like speed of a viscous fluid traveling through a pipe decreases as it moves away from the center of the pipe. If the center of the sphere is positioned on the axis X, the sum of all external forces on the sphere equals to 0. However, if the sphere is translated by y away from X, there is more light diffracted on the hemisphere closer to X, resulting in a force which pulls the sphere towards the region of high intensity.

Here we assumed that refractive index of the sphere n_{sphere} is greater than refractive index n_{medium} of the medium. A sphere with $n_{\text{sphere}} < n_{\text{medium}}$ would be pushed away from the region of high light intensity.

This technique has been used to manipulate microscopic objects. It was developed in 1986 by Arthur Ashkin and is today commonly known as the optical tweezers. A lens is used to focus the light beam in a point along the axis X and therefore the intensity has a global maximum in a point, to which the orbs are pulled. Sometimes two intersecting laser beams are used, creating the region of highest intensity (the trap) in the vertex. An analogous force applies to atoms, with same basic characteristics [3]. In short: atoms are pulled towards regions of high light intensity, if the light is NOT in one of absorbtion frequencies.

This is also used in creation of so-called optical lattices, where three perpendicular laser standing waves are used to create an array of local minima, where

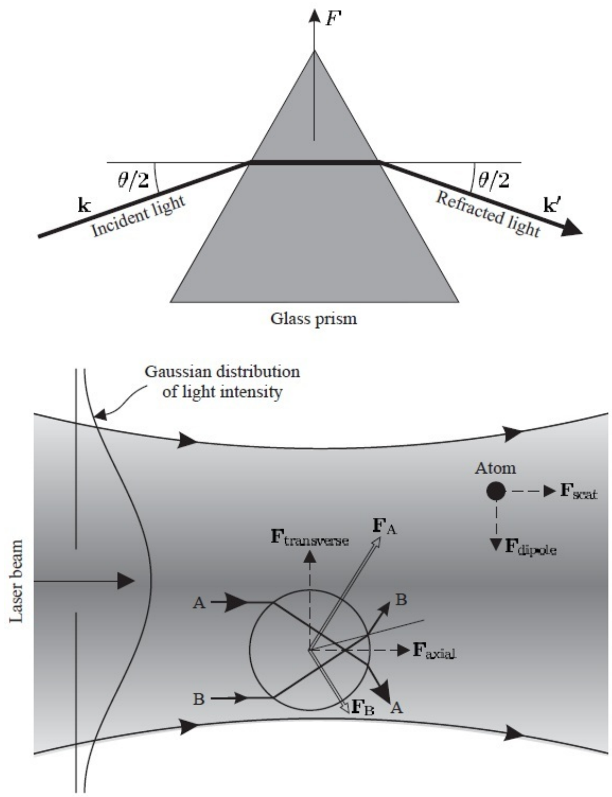


Figure 6: The dipole force. The sphere is pushed in the center of the laser beam, analogously the atom in the top right corner. F_{scatt} - the scattering force is pushing the atom along the laser beam, F_{dipole} - the dipole force is pulling it towards the center of high intensity.

atoms are put. Since the lasers are tunable, it is possible to simulate almost any potential and therefore simulate any matter.

Historically, this part of the cooling process wasn't done with dipole, but with magnetic trapping. The idea is similar, but instead of a light beam to create a local minimum in atom potential, a magnetic field was used.

6 Evaporative cooling

Optical molasses technique produces atoms well below the Doppler limit, but still considerably above recoil limit - the change of velocity that an atom receives when absorbing or emitting a single photon - and required temperature for achieving BEC at current atom density. Evaporative cooling, the last stage in achieving BEC is actually the extension of dipole trapping.

Using the dipole light force, we can create a finite harmonic potential, by creating the right laser beam. This potential is finite, and allows for atoms with above average kinetic energy to break free and escape the trap. However in doing so each runaway atom takes with him above average kinetic energy, effectively cooling the remaining atoms. After the most energetic particles are removed from the system the boundary is slightly lowered and a new bunch of atoms is allowed to escape, again cooling the remaining ones. This process can be repeated indefinitely and does not have a theoretical cooling limit. It is however a trade-off between the number of atoms lost and lowest temperature reached.

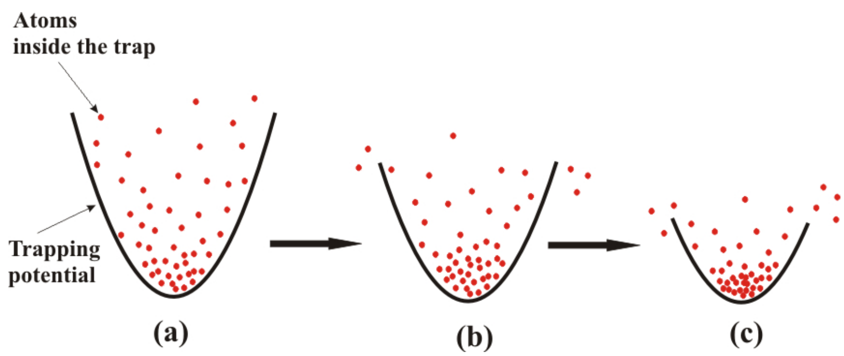


Figure 7: Evaporative cooling. Atoms inside the trap can escape, if they have enough kinetic energy, effectively cooling the remaining ones. By lowering the borders we speed up the process, and succeed in creating BEC.

When critical temperature (depending on what element is cooled) is reached, the particles collapse to ground state and Bose-Einstein condensate is created.

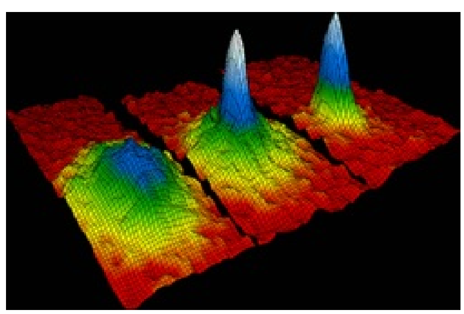
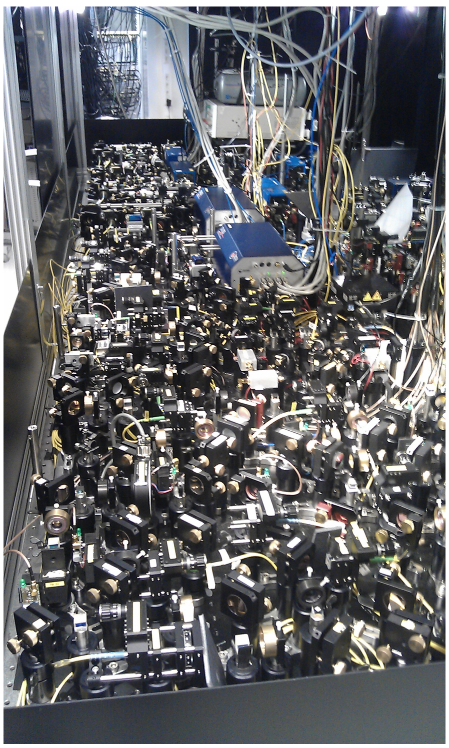
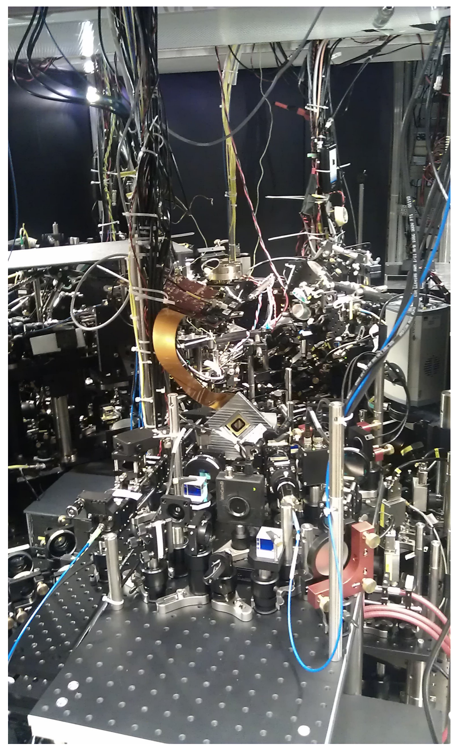


Figure 8: The distribution of velocities in a BEC. Adopted from [**coloradoedu**]

7 The experiment

While theoretical background is fairly straightforward, practical implementation is not. Producing good enough laser light in itself is a difficult task, that requires the use of hundreds of optical elements as seen below, while the chamber with BEC suffers from a lack of space. A typical BEC system uses up to 9 lasers, 2 coils, the slowing solenoid, a CCD camera for observing what is going on, and additional components needed for the experiment. Until this day several different bosons were successfully condensed, mostly alkali metals and alkali earths.





8 Conclusion

By using techniques of atomic beam slowing, optical molasses, magneto-optical trap and evaporative cooling the scientists have succeeded in creating the world's coolest material. Even if difficult, the unique properties of BEC make it a fascinating subject to research. In 2007, N. Ginsberg, S. Garner and L. Hau managed to send a light pulse to a BEC, where it was transformed into an atomic pulse, which traveled to the second BEC where it revived the original light beam with the same characteristics [4]. In 2009, R. Zhang, S. Garner and L. Hau published an article in which they described how they managed to store light in a BEC for more than 1 s, effectively reducing it to the speed of 25 km/h [7]. C. Weitenberg and others have, in 2011, used a tightly focused laser beam with a microwave field to flip the spins of individual atoms in an atomic Mott insulator [6]. These are just a few of numerous experiments, that have been made possible by using Bose-Einstein condensate in atomic gasses.

I believe that, in time, Bose-Einstein condensate will prove to be a major milestone in our understanding of the universe we live in.

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