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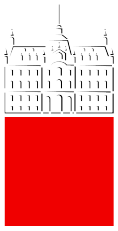

Synthetic electromagnetic fields for ultracold atoms

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Abstract

In the seminar, we describe a technique to engineer a light-induced vector potential in the cold atoms. We outline the derivation of the Hamilton function, which describes the atom, coupled with laser light, and introduce the effects that are cruicial for the creation of synthetic potential. We introduce the experimental setup needed for the synthetic vector potential, which generates non-zero synthetic magnetic field in the cold atoms and derive the potential analiticaly. At last, we briefly review some experimental details.

Contents

1 Introduction

In 1995, the first atomic Bose-Einstein condensate (BEC) was experimentally achieved by Eric Cornell and Carl Wiemann, when they cooled the gas of rubidium atoms to 170 nK. In the same year, the first sodium BEC was made by Wolfgang Ketterle. For these achievements, they together received the 2001 Nobel prize for Physics [1]. Since the early years of creating ultracold gases in the BEC phase, the so-called ultracold physics has undergone a lot of development. Ultracold atoms are especially useful in the studies of many-body correlated states, because the experimental setup allows the observed system to be widely tunable and nearly disorder-free [2]. The many-body phenomena, which are observed in condensed matter systems can be experimentally realized with ultracold atoms and then quantitatively studied. Effects that have been simulated in this way include, among others, the quantum Hall effect and spin-orbit coupling [3].

Ultracold systems usually contain between a few hundred and a few hundred million atoms at the temperatures between 1 nK and few μK. The atoms which are used in ultracold experiments are neutral and that represents an obstacle in the simulation of phenomena which include charged particles that obey the Lorentz force. But if we artificially generate the gauge field, similar to electromagnetic field, we can force the atoms to act as though as if they would have a non-zero charge. In the following chapters, we will introduce the creation of light induced gauge fields in the ultracold systems, from both the theoretical and the experimental point of view. We will, however, completely skip the description of the cooling process, by the means of which the Bose-Einstein condensate is created — the information about the cooling procedures can be found in [4].

2 Theoretical background

2.1 The aim of the experiment

The Hamiltonian of a charged particle with mass m and charge q in the presence of electromagnetic field is [5]:

$$H = \frac{1}{2m}(\boldsymbol{p}(t) - qA(\boldsymbol{r},t))^2 + q\varphi(\boldsymbol{r},t),$$

where $\boldsymbol{A}$ is the vector potential, φ is the scalar potential and $\boldsymbol{p}$ represents the canonical momentum of the particle. The potentials are defined up to a (scalar) gauge function:

$$\boldsymbol{A} \to \boldsymbol{A} + \boldsymbol{\nabla}\xi, \quad \varphi \to \varphi - \frac{\partial \xi}{\partial t}.$$

Magnetic and electric fields are dependent on the vector and scalar potentials in the following way:

$$\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A}, \quad \boldsymbol{E} = -\boldsymbol{\nabla}\varphi - \frac{\partial \boldsymbol{A}}{\partial t}. \tag{1}$$

To produce synthetic electromagnetic fields for neutral atoms, we must apply an artificially produced vector potential to the atoms. The aim of the experiment is to artificially produce the gauge field, i.e. the vector (scalar) potential. In the presence of artificially generated gauge field, neutral ultracold atoms act in the same way as a charged particle would act in the presence of magnetic (electric) field. This will be achieved by exposing the ultracold atoms to the magnetic field $\boldsymbol{B}$ and illuminating them by counterpropagating laser beams.

To properly explain the process of creating an artificial gauge field, we will first construct a Hamiltonian of the atom which is placed in the magnetic field (Ch. 2.2). We will continue with the description of the Stark Effect (Ch. 2.3) which — to some extent — describes the atom-light coupling in atoms. After that, we will take a look at the laser setup (Ch. 2.4) and explain the light-atom coupling (Ch. 2.5), which results in the effective vector potential, that produces non-zero effective magnetic field in ultracold atomic systems (Ch. 2.6 and 2.7).

2.2 The atom in the magnetic field

In order to successfully describe the interaction of atoms with the laser light, we must first construct the Hamiltonian of an atom in an external magnetic field. The atomic nucleus has a magnetic moment μ_I, which depends on the nuclear spin I. The atomic electrons therefore create the magnetic field $\boldsymbol{B}_e$ at the position of the atomic core. The interaction of the magnetic moment with B_e is described by the Hamiltonian [4]:

$$H_{HFS} = -\boldsymbol{\mu}_I \cdot \boldsymbol{B}_e = A\check{\boldsymbol{I}} \cdot \check{\boldsymbol{J}}, \tag{2}$$

where $\check{\boldsymbol{I}}$ represents the nuclear angular momentum and $\check{\boldsymbol{J}}$ represents the sum of the orbital and electronic spin angular momentum. This interaction gives rise to hyperfine atomic structure (Fig. 1)

In the presence of the external magnetic field $\boldsymbol{B}$, the Zeeman effect occurs. The contribution of the Zeeman effect to the total Hamiltonian of the atom equals:

$$H_{ZE} = \frac{\mu_B}{\hbar}\boldsymbol{B} \cdot (g_J\check{\boldsymbol{J}} + g_I\check{\boldsymbol{I}}) = \frac{\mu_B}{\hbar} g_F \boldsymbol{B} \cdot \check{\boldsymbol{F}}. \tag{3}$$

In the above equation, we introduced the total angular momentum $\check{\boldsymbol{F}} = \check{\boldsymbol{J}} + \check{\boldsymbol{I}}$ and the corresponding Landé g-factor:

$$g_F = g_J \frac{(F+1) - I(I+1) + J(J+1)}{2F(F+1)}. \tag{4}$$

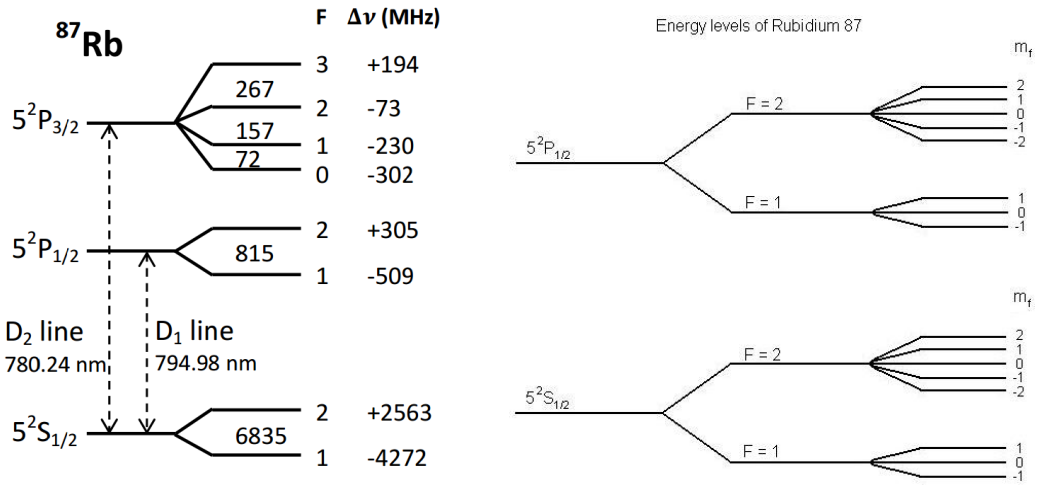

Figure 1: [Left] The fine structure (the splitting into $P_{1/2}$ and $P_{3/2}$ levels) of ^{87}Rb [10] arises as a result of spin-orbit interaction [4]. The transitions from the state $5^2S_{1/2}$ into the excited states are denoted by D_1 and D_2 transition lines. [Right] The hyperfine structure of $5^2S_{1/2}$ and $5^2P_{1/2}$ in ^{87}Rb [11].

We must also take into consideration the kinetic energy of the atom, $H_K = \boldsymbol{p}^2/2m$. The total Hamiltonian of the atom in the external magnetic field thus equals:

$$H_0 = \frac{\boldsymbol{p}^2}{2m} + \frac{\mu_B}{\hbar} g_F \boldsymbol{B} \cdot \check{\boldsymbol{F}} + A \check{\boldsymbol{I}} \cdot \check{\boldsymbol{J}}. \tag{5}$$

Later on, we will add another term to the atomic Hamiltonian to include the effect of the coupling between the light and the atomic internal states, which occurs when the atoms are illuminated by laser beams. But first, we will take a closer look at the Stark effect, which will give us some insight into the atom-light coupling.

2.3 The Stark effect

In the following chapter, we will discuss the effect of electromagnetic radiation on the atomic energy levels. We will treat the atom as a two-level system with wavefunction

$$\Psi(r,t) = c_1 \left|1\right\rangle e^{-iE_1 t/\hbar} + c_2 \left|2\right\rangle e^{-iE_2 t/\hbar}, \tag{6}$$

where E_1 and E_2 are the eigenenergies of the ground and the excited state. The probability that the atom occupies a certain state equals $|c_1|^2$ for the ground and $|c_2|^2$ for the excited state, $|c_1|^2 + |c_2|^2 = 1$. We assume that the majority of the atomic population is in the ground state in the beginning of the experiment, $c_1 = 1$, $c_2 = 0$.

The electromagnetic radiation perturbs the eigenfunctions of the atom [4]. The Hamiltonian of the system consists of two parts, $H = H_0 + H_1(t)$. The eigenvalues of the time-independent Hamiltonian describe energies of the atom in the absence of electric field. The time-dependent part of the Hamiltonian describes interaction with the radiation:

$$H_0 \Psi_n = E_n \Psi_n, \qquad H(t) = e\boldsymbol{r} \cdot \boldsymbol{E}_0 \cos(\omega t), \tag{7}$$

where $e\boldsymbol{r}$ denotes the electric dipole moment and $\boldsymbol{E}_0$ is the amplitude of the electric field which oscillates with the frequency ω. We insert the above Hamiltonian into the Schrödinger equation. Defining $\omega_0 = (E_2 - E_1)/\hbar$, we obtain the following two differential equations:

$$i\dot{c}_1 = c_2 \cos(\omega t) e^{-i\omega_0 t}\Omega = \frac{c_2}{2}\{e^{i(\omega-\omega_0)t} + e^{-i(\omega+\omega_0)t}\}\Omega, \tag{8}$$

$$i\dot{c}_2 = c_1 \cos(\omega t) e^{i\omega_0 t}\Omega^* = \frac{c_1}{2}\{e^{-i(\omega-\omega_0)t} + e^{i(\omega+\omega_0)t}\}\Omega^*. \tag{9}$$

In these equations, Ω represents the Rabi frequency, which is proportional to the amplitude of the electric field:

$$\Omega =< 1|r|2 > \frac{e|\boldsymbol{E_0}|}{\hbar}.$$

We can simplify the Eq. (8) and Eq. (9) by making the rotating-wave approximation [4]. We assume that the atoms are illuminated with the monochromatic laser beams with frequency ω, which is detuned from frequency ω_0:

$$\omega = \omega_0 + \delta = \frac{(E_2 - E_1)}{\hbar} + \delta, \qquad |\delta| \ll \omega_0$$

In the above equation, δ represents the detuning, i.e. the shift from the resonance frequency ω_0. Since the detuning is small in comparison to ω_0, $\omega - \omega_0 \ll \omega + \omega_0$ and therefore the second terms in Eq. (8) and Eq. (9) oscillate much faster that the first ones. The contribution of the second terms thus averages to zero in the typical light-atom interaction time. We introduce new variables

$$\tilde{c}_1 = c_1 e^{-i\delta t/2}, \qquad \tilde{c}_2 = c_2 e^{i\delta t/2}. \tag{10}$$

The new variables still contain the information about population of both states, because $|\tilde{c}_1|^2 = |c_1|^2$ (this is also true for c_2). We insert the new variables into the Eq. (8) and Eq. (9) and then rewrite them in the matrix form:

$$i\frac{d}{dt}\begin{pmatrix} \tilde{c}_1 \\ \tilde{c}_2 \end{pmatrix} = \begin{pmatrix} \delta/2 & \Omega/2 \\ \Omega/2 & -\delta/2 \end{pmatrix}\begin{pmatrix} \tilde{c}_1 \\ \tilde{c}_2 \end{pmatrix}. \tag{11}$$

The solutions of the above system have the form:

$$\begin{pmatrix} \tilde{c}_1 \\ \tilde{c}_2 \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} e^{-i\lambda t}, \tag{12}$$

where λ are the eigenvalues of the matrix from the Eq. (11). We're only interested in solutions in cases where the frequency detuning is large, $|\delta| \gg \Omega$. In that case, the eigenvalues are:

$$\lambda \simeq \pm(\frac{\delta}{2} + \frac{\Omega^2}{4\delta}). \tag{13}$$

If the detuning is large enough, the absorbtion of the photons by atoms is hardly present and therefore we don't have to include the effect of absorption into our description of the light shift. By combining the Eq. (10), (12) and (13) we obtain the new values of coefficients c_1 and c_2 and after we insert them into equation (6), we get:

$$\Psi(r,t) = a\,|1\rangle\, e^{-i(E_1/\hbar - \Omega^2/4\delta)t} + b\,|2\rangle\, e^{-i(E_2/\hbar + \Omega^2/4\delta)t}. \tag{14}$$

We see that the eigenenergies have changed because of the atom-light interaction (Fig. 2). The energy change is proportional to the intensity of the laser beam $|\boldsymbol{E}|^2$.

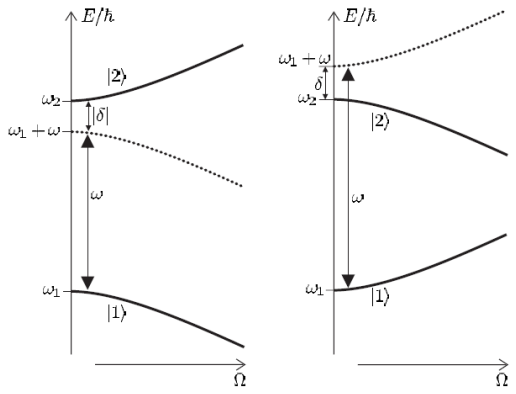

Figure 2: Change in eigenenergies due to interaction with electromagnetic field [4]. In the left picture, the detuning is positive and in the right picture, the detuning is negative.

2.4 Illumination of atoms with laser light

In order to synthetisize the gauge potential, we illuminate the atoms with two laser beams. The laser beams are counter propagating, their frequencies differ by the detuning $\delta\omega$ and their polarizations are orthogonal (see Fig. 3). In this case:

$$\boldsymbol{E}_{\omega_-} = E\exp(ik_R x - i\omega t)\boldsymbol{e}_y, \qquad \boldsymbol{E}_{\omega_+} = E\exp(-ik_R x - i(\omega+\delta\omega)t)\boldsymbol{e}_z. \tag{15}$$

The laser beam frequencies differ by (see Fig. 4):

$$\delta\omega = g_F\mu_B|\boldsymbol{B}|/\hbar - \delta, \qquad |\delta| \ll \delta\omega.$$

The experiment which we are going to describe takes place in the presence of the magnetic field $\boldsymbol{B}{=}B\boldsymbol{e}_z$. The first term on the right side of the equation represents the linear Zeeman shift between different m_F levels.

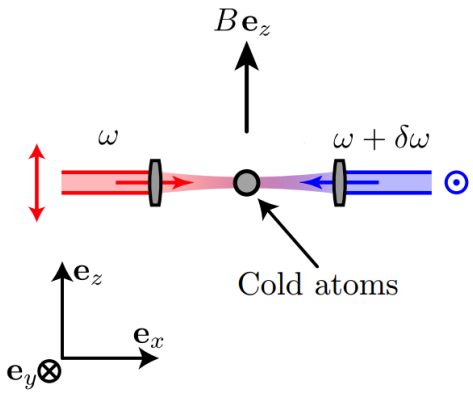

Figure 3: Two linearly polarised laser beams are used to illuminate the cold atoms in the magnetic field $\boldsymbol{B}$ [3].

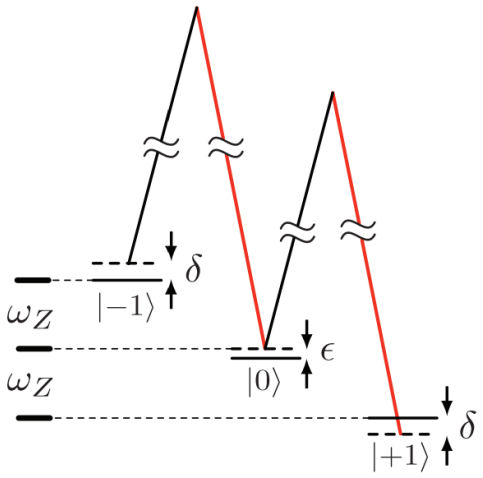

Figure 4: The transitions between the atomic states occur because of the Raman coupling, which involves absorption (black line) and stimulated emission (red line) of a photon.

2.5 Light-atom coupling

We have shown in Chapter 2.3 that the eigenenergies of a two-level atom change if we illuminate the atom with the laser light (Eq. (14)). We have also pointed out that the change in the energy is proportional to the square of the amplitude of the laser light $|\boldsymbol{E}|^2$. The experiments in which electromagnetic fields are synthetized are carried out on alkali atoms. Because of that, we must take into account the spin-orbit coupling contribution to the energy [6]. Because of that, the atom-light coupling Hamiltonian for alkali atoms is more complicated.

The atom-light coupling Hamiltonian for alkali atoms in the ground state takes the form

$$H_L = [u_s(\boldsymbol{E}^* \cdot \boldsymbol{E}) + \frac{iu_v(\boldsymbol{E}^* \times \boldsymbol{E})}{\hbar} \cdot \check{\boldsymbol{J}}], \tag{16}$$

where u_s is proportional to the atoms AC polarizability and $u_v = -2u_s\Delta_{FS}/3(\omega - \omega_0)$, where Δ_{FS} denotes the fine-structure splitting, which is proportional to the difference between D_1 and D_2 transition energies (Fig. 1). In the context of this seminar, the values of u_s and u_v are not important and we will not discuss them any further. The second term in the Eq. (16), i.e. "the vector term", acts on the momentum $\check{\boldsymbol{J}}$ in the same way as the magnetic field acts on the total angular momentum $\check{\boldsymbol{F}}$ in the Eq. (5). Because of this similarity with the interaction with magnetic field, the contribution of the vector light shift to the energy of the atom can be seen as the effective magnetic field:

$$\boldsymbol{B}_{eff} = \frac{iu_v(\boldsymbol{E}^* \times \boldsymbol{E})}{\mu_B g_J}. \tag{17}$$

When we add the light-atom coupling contribution (16) to the Hamiltonian (5) we obtain the total Hamiltonian of the ground state atom, which is exposed to the external magnetic field and to the optical electric field:

$$H = \frac{\boldsymbol{p}^2}{2m} + u_s\boldsymbol{E}^* \cdot \boldsymbol{E} + \frac{\mu_B g_F}{\hbar}(\boldsymbol{B} + \boldsymbol{B}_{eff}) \cdot \check{\boldsymbol{F}} + A_{hf}\check{\boldsymbol{J}} \cdot \check{\boldsymbol{I}} - \frac{\mu_B g_I}{\hbar}\boldsymbol{B}_{eff} \cdot \check{\boldsymbol{I}}. \tag{18}$$

If we are focusing on alkali atoms, we can safely neglect the last term in the Hamiltonian, because the ratio $|g_I/g_J| \approx 0.0005$ is very small [3]. Furthermore, we do not need to take the term $A_{hf}\check{\boldsymbol{J}} \cdot \check{\boldsymbol{I}}$

into account, because its value is constant if all atoms are in a single hyperfine state with fixed f[1]. From now on, we will therefore only refer to the first three terms of the above Hamiltonian, i.e. the kinetic, the scalar and the vector term.

We will now describe the interaction of the atoms with the Hamiltonian (18) and the laser beams, described by Eq.(15). In this experimental setup, the effective magnetic field is non-zero and both the scalar and the vector light shift are present.

The Hamiltonian (18) contains time-dependent terms, which can be eliminated [6] if the unitary transformation $\hat{S} = \exp(-i\delta\omega t\hat{F}_z)$ is applied. After that, the Hamiltonian can be simplified if we time-average to zero all explicitly time-dependent terms that emerge in the Hamiltonian when the AC electric field (15) is applied. This approximation is justified by the fact that the typical light-atom interaction time is much larger than the oscillation time of time-dependent terms in the Hamiltonian. In other words, we make the rotating-wave approximation, which we have already introduced in Chapter 2.3. However, this approximation is only possible if $|B| \gg |B_{eff}|$.

After we make the rotating wave approximation, the Hamiltonian takes the form:

$$H = \frac{p^2}{2m} + U(\boldsymbol{r})\check{1} + \boldsymbol{\Omega} \cdot \check{\boldsymbol{F}}. \tag{19}$$

The kinetic part is the same as before. The term containing the scalar potential $U(\boldsymbol{r})$ and the identity matrix describes a constant scalar light shift due to atom-light interaction. The last term represents the coupling between the electrical field and angular momentum; this time, the effective magnetic field is represented by the so-called Zeeman vector $\boldsymbol{\Omega}$, which differs from $\boldsymbol{B}_{eff}$ because it no longer includes the time-dependent terms which we eliminated with the rotating-wave approximation. In our particular case, due to interaction with laser light (15), the scalar potential $U(\boldsymbol{r})$ and the effective magnetic field Ω take the following forms:

$$U(\boldsymbol{r}) = 2u_s E^2, \tag{20}$$

$$\boldsymbol{\Omega} = \delta \boldsymbol{e}_z + \Omega_R(\sin(2k_R x)\boldsymbol{e}_x - \cos(2k_R x)\boldsymbol{e}_y), \tag{21}$$

where $\Omega_R = u_v E^2/\hbar$. The single photon recoil wave vector is represented by $k_R = 2\pi/\lambda$, where and recoil energy is $E_R = \hbar^2 k_R^2/2m$. We have now obtained the Hamiltonian of the atom coupled with the laser light. In the next chapter, we will perform a set of unitary transformations on the Hamiltonian in order to prove the existence of the effective vector potential, which emerges as a consequence of the atom-light coupling in ultracold atoms.

2.6 Effective vector potential

In general, the atom-light coupling term of Hamiltonian (19) equals:

$$\hat{M} = \boldsymbol{\Omega} \cdot \check{\boldsymbol{F}} = |\boldsymbol{\Omega}|(\cos\theta \check{F}_z + \sin\theta\cos\phi \check{F}_x + \sin\theta\sin\phi \check{F}_y). \tag{22}$$

The coupling is described by the operator $\hat{M}$, which encompasses (x, y, z) components of both the operator $\check{\boldsymbol{F}}$ and Zeeman vector $\boldsymbol{\Omega}$, where $|\boldsymbol{\Omega}| = \sqrt{\delta^2 + \Omega_R^2}$. The definition of the angles is included in the Fig. 5.

[1] This is true in the case of ultracold atoms[6].

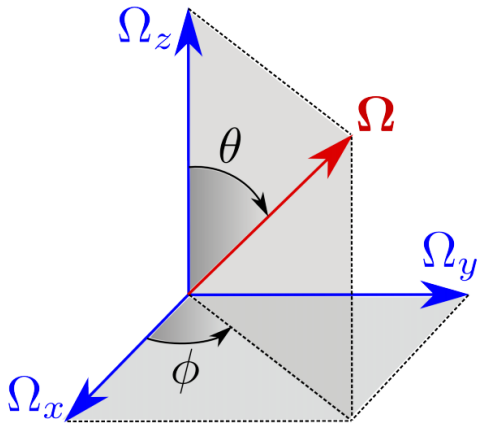

Figure 5: The angles between Ω and its components are defined as ϕ and θ.

We want to introduce the new coordinate system, in which the coupling vector $\mathbf{\Omega}$ will point along z axis. We achieve this by the means of unitary transformation

$$\hat{R} = e^{-i\check{F}_z\phi/\hbar} e^{-i\check{F}_y\theta/\hbar} e^{i\check{F}_z\phi/\hbar}. \tag{23}$$

If we take a look at the Eq. (21), we see that the appropriate angles for our desired rotation are:

$$\phi = 2k_R x - \frac{\pi}{2}, \qquad \theta = \arccos(\frac{\delta}{\Omega}).$$

After the transformation, the Eq. (22) takes the form

$$\hat{R}^\dagger \hat{M} \hat{R} = |\mathbf{\Omega}| \check{F}_z. \tag{24}$$

After transforming the whole Hamiltonian (19) with the transformation (23), we obtain

$$\hat{H} = \frac{\hbar^2}{2m}(\hat{k} - \frac{\hat{\boldsymbol{A}}}{\hbar})^2 + U(\boldsymbol{r})\check{1} + \sqrt{\delta^2 + \Omega_R^2}\check{F}_z, \tag{25}$$

where

$$\hat{\boldsymbol{A}}(\boldsymbol{r}) = \frac{2k_R(\delta\check{F}_z - \Omega_R\check{F}_x)}{\sqrt{\delta^2 + \Omega_R^2}}\boldsymbol{e}_x + \check{F}_y \boldsymbol{\nabla}_r \theta_y. \tag{26}$$

It should be noted at this point that the transformation of the Hamiltonian (19) into the form (25) has no consequences from the physical point of view [9]. The transformation only changes the basis in which the wavefunctions are represented. In the initial basis, no vector potential was present in the Hamiltonian (19), but after the transformation, the potential $\hat{\boldsymbol{A}}$ emerged.

2.7 Dressed states and adiabatic approximation

Because the operator $\check{F}$ obeys the angular momentum algebra [11], we know that the eigenvalues of the operator $\check{F}_z$ are:

$$\check{F}_z \left|m_F\right\rangle = \hbar m_F \left|m_F\right\rangle. \tag{27}$$

The eigenstates of Hamiltonian (19) are called "dressed states". Using the unitary transformations, we rotatated the Hamiltonian into the form described by (25). The eigenstates of the rotated

Hamiltonian are "bare" atomic states $|m_F\rangle$. Let us briefly return to the light-atom coupling matrix $\hat{M}$, introduced in the previous chapter. The Eq. (24) shows the coupling matrix, which was rotated into the basis with eigenstates $|m_F\rangle$. From the Eq. (27) it follows that the eigenvalues of $\hat{M}$ equal:

$$V_{m_F} = \hbar m_F \Omega \tag{28}$$

As we have mentioned in the introduction, the synthetization, i.e. the artifical generation of the electromagnetic fields, is carried out on ultracold atoms in the BEC phase. In the non-BEC atomic population, the atoms are either in excited states or in the ground state. But below the critical temperature, at which the transition into the BEC phase takes place, all the condensed atoms are in the ground state [8]. In the case of ^{87}Rb BEC atoms, the angular momentum of the ground level is $F = 1$ and the lowest energy state is $m_F = -1$ (see Fig. 1).

The ground state $m_F = -1$ and the first excited state $m_F = 0$ are separated by the splitting $\hbar\Omega = \hbar\sqrt{\Omega_R^2 + \delta^2}$, according to Eq. (28). If we ensure that the energy gap between the two levels is larger that typical kinetic energy of the atom, $\hbar\Omega_R \gg E_R$, the atoms can not gather enough energy to jump into excited state and therefore remain in the ground state. This is called the adiabatic or the Born-Oppenheimer approximation.

As we have pointed out in Eq. (24), the rotated matrix $\hat{M}$ equals $\Omega\check{F}_z$ and therefore the eigenfuctions of $\hat{M}$ are functions $|m_F\rangle$. Since we applied the Born-Oppenheimer approximation, we can now obtain the Hamiltonian of the atom in the dressed state simply by projecting it to the only state which has the non-zero population, $m_F = -F$. After we make the projections, $\left\langle m_F \left| \check{F}_x \right| m_F \right\rangle = \left\langle m_F \left| \check{F}_y \right| m_F \right\rangle = 0$ and $\left\langle m_F \left| \check{F}_z \right| m_F \right\rangle = \hbar m_F$, we obtain the vector potential:

$$\boldsymbol{A}_{\boldsymbol{m_F}}(\hat{\boldsymbol{r}}) = 2\hbar k_R m_F \left(\frac{\delta}{\sqrt{\delta^2 + \Omega_R^2}} \right) \boldsymbol{e}_x. \tag{29}$$

Contrary to the effective vector potential (26), which generated no synthetic magnetic field, the magnetic field associated with the vector potential $\boldsymbol{A}_{m_F}$ is not necessarily zero:

$$\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A}_{m_F} = 2\hbar k_R m_F (\boldsymbol{e}_y \partial_z - \boldsymbol{e}_z \partial_y) \left(\frac{\delta}{\sqrt{\delta^2 + \Omega_R^2}} \right).$$

The above equation reveals that, if we want to generate a non-zero synthetic magnetic field, we must make sure that either δ or Ω depend on either y or z.

3 Experimental implementation

In the preceeding chapter, we have shown that the atom-light interactions in the ultracold atoms can result in the vector potentials, which generate synthetic magnetic fields. The resulting vector potential (29) is pointing along the axis x and it does not generate any magnetic field, unless the detuning or the optical intensity varies linearly along y or z. Instead of more in-depth treatment of the solving of the Eq. (19) [3], we will now focus on the experimental technique, which is used to synthetisize the vector potentials and, consequently, the electric and magnetic fields [12][13]. We will describe the experiment which was made on the ^{87}Rb atoms.

3.1 The synthetic vector potential

Before the Raman laser beams are turned on, the ^{87}Rb atoms are cooled down to extremely low temperatures into the Bose-Einstein condensate (BEC). When the atomic population is in the BEC

phase, the majority of atoms occupies the lowest energy level, i.e. the ground state. In this case, total angular momentum quantum number equals $F = 1$ (see Fig. 1) and corresponding internal states are $m_F \in \{1, 0, -1\}$. In the following paragraphs, only the internal angular momentum $\hbar m_F$ and linear momentum $\hbar k_x$ will be important. In our notation, an atomic state will therefore be denoted by $|m_F, k_x >$.

Once the temperature of the atoms is low enough and the atoms are in BEC state, the fields can be synthetisized using Raman lasers. Before the Raman lasers are turned on, the atoms occupy the lowest energy state $|m_F = -1, k_x = 0 >$. The laser setup is the same as shown on Fig. 3. Once the lasers are turned on, the light couples together all three spin states $m_F \in \{-1, 0, 1\}$. The three coupled atomic states with the same quasimomentum $\tilde{k}_x$ are

$$| - 1, \tilde{k}_x + 2k_r >, \quad |0, \tilde{k}_x >, \quad | + 1, \tilde{k}_x - 2k_r > . \tag{30}$$

The momenta of two states with $\Delta m_F = 1$, differ by $2k_R$, because in the case of Raman coupling, atom absorbs one photon which is then emitted in the opposite direction, when the atom changes states (Fig. 4). The change of atomic momentum due to absorption is $\hbar k_R$ and the emission contributes $\hbar k_R$ as well, because the direction of flight of the spontaneously emitted photon is the opposite to the direction of flight of absorbed photon. This leads to three dressed states with the energy-momentum dispersions which are illustrated in Fig. 6.

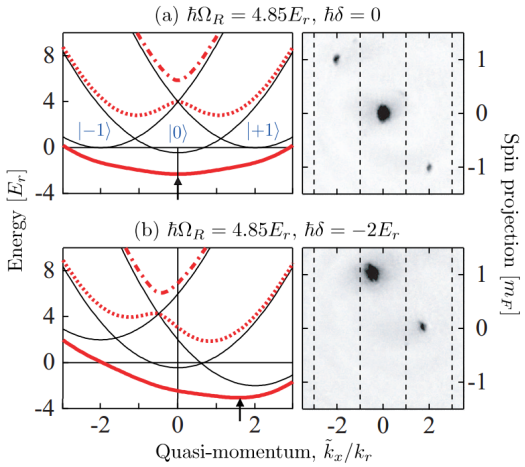

Figure 6: [Left] Computed energy-momentum dispersion curves. [Right] Raman-dressed state; horizontal axis represents the momentum and vertical axis represents the m_F components of the dressed state.

The states, described by Eq. (30) in the absence of the Raman coupling are represented with black curves in the left part of Fig. 6. The red curves represent Raman-coupled dressed states. If the frequency offset in the laser frequencies is not detuned from the resonance frequency, $\delta = 0$, the effective vector potential is zero (see Eq. (29)). If the detuning is slowly turned on, the atoms remain in the minimum of momentum-dependent energy, because they adiabatically follow the ground dressed state (see Ch. 2.7). The minimum of the energy is shifted when the detuning is slowly changed to a non-zero value, which indicates the non-zero effective vector potential.

The right part of Fig. 6 shows that the dressed state consists of three different atomic m_F states with corresponding momenta $k_{m_F} = \tilde{k}_x - 2m_F k_R$, when the detuning is zero. Once the detuning is changed slowly to a non-zero value, the displacement of momentum of the $m_F = 0$ state from zero again shows that the effective vector potential is present.

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